Chapter 3

ECONOMIC AND ECONOMETRIC MODELS

MICHAEL D. INTRILIGATOR*

University of California, Los Angeles

Contents

*The author acknowledges, with appreciation, the helpful suggestions of Ray Fair, Gary Fromm, Zvi Griliches, Elizabeth Landaw, and Henri Theil.

Handbook of Econometrics, Volume I, Edited by Z. Griliches and M.D. Intriligator

1. Introduction and overview

This chapter of the Handbook will present a discussion of models, particularly models used in econometrics.[1] Models play a major role in all econometric studies, whether theoretical or applied. Indeed, defining *econometrics* as the branch of economics concerned with the empirical estimation of economic relationships, models, together with data, represent the basic ingredients of any econometric study. Typically, the theory of the phenomena under investigation is developed into a model which is further refined into an econometric model. This model is then estimated on the basis of data pertaining to the phenomena under investigation using econometric techniques. The estimated model can then be used for various purposes, including structural analysis, forecasting, and policy evaluation.

This chapter provides a discussion of models and economic models in Section 2, and comparative statics in Section 3. Section 4 then presents econometric models, including the structural form, reduced form, and final form. The problem of identification, which is presented in more detail in Chapter 4 of this Handbook, by Cheng Hsiao, is discussed in Section 5. Section 6 provides some examples of specific models, including demand (discussed in more detail in Chapter 30 of this Handbook by Angus Deaton), production (discussed in more detail in Chapter 31 of this Handbook by Dale Jorgenson), macroeconometric models (also discussed in Chapters 33, 34, and 35 of the Handbook by Ray Fair, John Taylor, and Lawrence Klein, respectively), and other econometric models. Section 7 presents a discussion of the uses of econometric models, specifically structural analysis, forecasting (further discussed in Chapter 33 of this Handbook by Ray Fair), and policy evaluation (further discussed in Chapters 34 and 35 of this Handbook by John Taylor and Lawrence Klein, respectively). Section 8 presents a conclusion.

2. Models and economic models

A *model* is a simplified representation of an actual phenomenon, such as an actual system or process. The actual phenomenon is represented by the model in order to explain it, to predict it, and to control it, goals corresponding to the three

[1]This chapter is based to a large extent on material presented in Intriligator (1978, esp. ch. 1, 2, 7, 8, 10, 12, 13, 14, 15, and 16). Other general references on economic and econometric models include Beach (1957), Suits (1963), Christ (1966), Bergstrom (1967), Ball (1968), Kendall (1968), Cramer (1969), Malinvaud (1970), Bridge (1971), Goldberger and Duncan (1973), Maddala (1977), Leamer (1978), Zellner (1979), and Arnold (1981). Other chapters in this Handbook that treat economic and econometric models include Chapter 4 by Hsiao, Chapter 5 by Leamer, Chapter 26 by Lau, Chapter 28 by Maddala, and Chapter 29 by Heckman and Singer.

purposes of econometrics, namely structural analysis, forecasting, and policy evaluation. Sometimes the actual system is called the *real-world system* in order to emphasize the distinction between it and the model system that represents it.

Modeling, that is, the art of model building, is an integral part of most sciences, whether physical or social, because the real-world systems under consideration typically are enormously complex. For example, both the motion of an elementary particle in an accelerator and the determination of national income are real-world phenomena of such complexity that they can be treated only by means of a simplified representation, that is, via a model. To be most useful a model has to strike a reasonable balance between realism and manageability. It should be realistic in incorporating the main elements of the phenomena being represented, specifying the interrelationships among the constituent elements of the system in a way that is sufficiently detailed and explicit so as to ensure that the study of the model will lead to insights concerning the real-world system. It should, however, at the same time be manageable in eliminating extraneous influences and simplifying processes so as to ensure that it yields insights or conclusions not obtainable from direct observation of the real-world system. The art of model building involves balancing the often competing goals of realism and manageability.

Typically the initial models of a phenomena are highly simplified, emphasizing manageability. They may, for example, model the system under study as a "black box", treating only its inputs and outputs without attempting to analyze how the two are related. Later models are typically more elaborate, tracking inputs forward and outputs backward until eventually an analytic model is developed which incorporates all the major interconnections between inputs and outputs in the real-world system. The process of modeling typically involves not only the analysis of interconnections between inputs and outputs but also the treatment of additional or related phenomena and greater disaggregation.

Many different types of models have been used in economics and other social and physical sciences. Among the most important types are verbal/logical models, physical models, geometric models, and algebraic models, involving alternative ways of representing the real-world system.

Verbal/logical models use verbal analogies, sometimes called *paradigms*, to represent phenomena. In economics two of the earliest and still two of the best paradigms were developed by Adam Smith.[2] The first was the pin factory, used by Smith as a model of the concept of division of labor. This concept is applicable at the national and international level, but the participants and processes become so numerous and their interrelations so manifold that the principle could be lost. Smith therefore used the paradigm of the pin factory, where the principle could be readily understood. The second paradigm employed by Smith was that of the

[2]See Smith (1776).

"invisible hand", one of the most important contributions of economics to the study of social processes. Smith observed that in a decentralized economy the price system guides agents to ensure that their individual actions attain a coherent equilibrium for the economy as a whole, promoting the general welfare of society. Again a complex process, in this case that of all economic actions, was represented by a verbal model.

Physical models represent the real-world system by a physical entity. An example is a scale model of a physical object, such as a scaled-down model airframe for an airplane, which is tested in a wind tunnel or a scaled-up model of a protein molecule. Economic systems have also been studied with physical models, including hydraulic models in which flows of fluids represent monetary flows in the economy. The most important physical models of economic phenomena, however, are those relying upon electric circuits, using the modern analog computer.[3]

Geometric models use diagrams to show relationships among variables. Such models have played an important role in the development of economics. For example, the geometric model of price determination in a single isolated market, involving intersecting demand and supply curves, is a fundamental one in microeconomic theory. Similarly the geometric model of the determination of national income, e.g. via the IS–LM diagram, is a fundamental one in macroeconomic theory. Such models are useful in indicating the principal relationships among the major variables representing the phenomena under investigation, but, because of the limited number of dimensions available, it is necessary to restrict geometric models to a relatively few variables. To deal with more variables usually involves use of an algebraic model.

Algebraic models, which are the most important type of models for purposes of econometrics, represent a real-world system by means of algebraic relations which form a system of equations. The system of equations involves certain variables, called *endogenous variables*, which are the jointly dependent variables of the model and which are simultaneously determined by the system of equations. The system usually contains other variables, called *exogenous variables*, which are determined outside the system but which influence it by affecting the values of the endogenous variables. These variables affect the system but are not in turn affected *by* the system. The model also contains *parameters* which are generally estimated on the basis of the relevant data using econometric techniques.

The general algebraic model can be expressed as the following system of g independent and consistent (i.e. mutually compatible) equations in the g endogenous variables, $y_1, y_2, \dots, y_g$, the k exogenous (or lagged endogenous) variables,

[3]For applications of electronic analog models to economics, see Morehouse, Strotz and Horwitz (1950), Enke (1951), Strotz, McAnulty and Naines (1953), and Tustin (1953).

$x_1, x_2, \ldots, x_k$, and the m parameters, $\delta_1, \delta_2, \ldots, \delta_m$:

$$\begin{aligned}
&f^1(y_1, y_2, \ldots, y_g; x_1, x_2, \ldots, x_k; \delta_1, \delta_2, \ldots, \delta_m) = 0, \\
&f^2(y_1, y_2, \ldots, y_g; x_1, x_2, \ldots, x_k; \delta_1, \delta_2, \ldots, \delta_m) = 0, \\
&\vdots \\
&f^g(y_1, y_2, \ldots, y_g; x_1, x_2, \ldots, x_k; \delta_1, \delta_2, \ldots, \delta_m) = 0.
\end{aligned} \tag{2.1}$$

In vector notation the general algebraic model can be written

$$f(y, x, \delta) = 0, \tag{2.2}$$

where f is a column vector of g functions, y is a row vector of g endogenous variables, x is a row vector of k exogenous (or lagged endogenous) variables, δ is a row vector of m parameters, and 0 is a column vector of zeros.

Assuming the functions are differentiable and that the Jacobian matrix of first-order partial derivatives is non-singular at a particular point:

$$\left|\frac{\partial f}{\partial y}\right| = \begin{vmatrix} \frac{\partial f^1}{\partial y_1} & \frac{\partial f^1}{\partial y_2} \cdots & \frac{\partial f^1}{\partial y_g} \\ \frac{\partial f^2}{\partial y_1} & \frac{\partial f^2}{\partial y_2} \cdots & \frac{\partial f^2}{\partial y_g} \\ \vdots & & \\ \frac{\partial f^g}{\partial y_1} & \frac{\partial f^g}{\partial y_2} \cdots & \frac{\partial f^g}{\partial y_g} \end{vmatrix} \neq 0 \quad \text{at } (y, x), \tag{2.3}$$

the implicit function theorem implies that at this point it is possible to solve the system of equations (2.2) for the endogenous variables as differentiable functions of the exogenous variables and parameters:[4]

$$y = \phi(x, \delta), \tag{2.4}$$

where ϕ if a column vector of g functions.

A very simple example is the determination of price in an isolated market, where the equations for demand and supply are

$$\begin{aligned}
&q - D(p, x, \delta) = 0 \\
&q - S(p, x, \delta) = 0.
\end{aligned} \tag{2.5}$$

[4] For discussions of the implicit function theorem, see Rudin (1964), Apostol (1974), and Hoffman (1975).

Here q and p are quantity and price respectively; D and S are demand and supply functions respectively; and x and δ are vectors of exogenous variables and parameters, respectively. The Jacobian condition is

$$\begin{vmatrix} 1 & -\dfrac{\partial D}{\partial p} \\ 1 & -\dfrac{\partial S}{\partial p} \end{vmatrix} \neq 0, \tag{2.6}$$

which is met if the customary slope conditions

$$\frac{\partial D}{\partial p} < 0 \quad \text{and} \quad \frac{\partial S}{\partial p} > 0 \tag{2.7}$$

are satisfied. Then the system of equations (2.4) can be solved for (equilibrium) quantity and price:

$$q = q(x, \delta); \qquad p = p(x, \delta). \tag{2.8}$$

3. Comparative statics

The comparative statics technique is one of the most useful techniques in economic analysis.[5] It involves the comparison of two equilibrium points of a system of equations such as (2.2), describing the phenomena under consideration. The two equilibrium points typically involve equilibrium before and after displacement by a change in one of the parameters of the system of equations.

Consider system (2.2) for which the Jacobian condition in (2.3) is met so the system can be solved for the endogenous variables as in (2.4). Inserting these solutions into (2.2) yields the system of g identities:

$$f[\phi(x, \delta), x, \delta] \equiv 0. \tag{3.1}$$

Now consider the effect of a change in one of the exogenous variables or parameters, say x_j, on the equilibrium values of the variables.[6] Differentiating each of the identities in (3.1) with respect to x_j yields

$$\sum_{h=1}^{g} \frac{\partial f^l}{\partial y_h} \frac{\partial y_h}{\partial x_j} + \frac{\partial f^l}{\partial x_j} = 0 \qquad l = 1, 2, \ldots, g. \tag{3.2}$$

[5]For a general discussion of the theory of comparative statics, see Samuelson (1947), Intriligator (1971), Kalman and Intriligator (1973), Silberberg (1978), and Intriligator (1981).

[6]A similar approach would yield the effect of any parameter, say δ_q, on the equilibrium values of the variables.

Solving for the effect of a change in every x_j, for $j = 1,2,\dots,k$, on y_h yields, in matrix notation,

$$\frac{\partial y}{\partial x} = -\left(\frac{\partial f}{\partial y}\right)^{-1}\frac{\partial f}{\partial x}, \tag{3.3}$$

where the three matrices are

$$\frac{\partial y}{\partial x} = \left(\frac{\partial y_h}{\partial x_j}\right); \qquad \frac{\partial f}{\partial y} = \left(\frac{\partial f^l}{\partial y_h}\right); \qquad \frac{\partial f}{\partial x} = \left(\frac{\partial f^l}{\partial x_j}\right). \tag{3.4}$$

Eq. (3.3) expresses the change in the equilibrium levels of each of the endogenous variables as each of the exogenous variables changes. The effect of a change $\mathrm{d}x_j$ in any one exogenous variable on the equilibrium value of any of the endogenous variables $\mathrm{d}y_h$ is then given as

$$\mathrm{d}y_h = \frac{\partial y_h}{\partial x_j}\mathrm{d}x_j, \tag{3.5}$$

where $\partial y_h/\partial x_j$ is the hj element of the $\partial y/\partial x$ matrix in (3.3).

Restrictions on the signs or values of the derivatives in $\partial f/\partial y$ and $\partial f/\partial x$ in (3.3) often lead to comparable restrictions on the signs or values of the derivatives in $\partial y/\partial x$. These qualitative restrictions on the effects of exogenous variables on endogenous variables provide some of the most important results in the analysis of economic systems described by an algebraic model.[7]

4. Econometric models

Econometric models are generally algebraic models that are *stochastic* in including random variables (as opposed to *deterministic* models which do not include random variables). The random variables that are included, typically as additive stochastic disturbance terms, account in part for the omission of relevant variables, incorrect specification of the model, errors in measuring variables, etc. The general econometric model with additive stochastic disturbance terms can be written as the *non-linear structural form* system of g equations:

$$f(y,x,\delta) = \varepsilon, \tag{4.1}$$

[7]For a discussion of qualitative economics, involving an analysis of the sign or value restrictions on partial derivatives, see Samuelson (1947) and Quirk and Saposnik (1968). For a specific example of these qualitative restrictions see the discussion of Barten's fundamental matrix equation for consumption theory in Chapter 1 of this Handbook by Theil.

where ε is a vector of stochastic disturbance terms, one for each equation. This form is similar to (2.2) with the addition of disturbance terms in each equation where ε is a vector of stochastic disturbance terms. If the conditions of the implicit function theorem are met these equations can be solved for the endogenous variables as differentiable functions of the exogenous variables and parameters, with the stochastic disturbance terms included as additive error terms. The resulting *non-linear reduced form* is the system of g equations:

$$y=\phi(x,\delta)+u, \tag{4.2}$$

where u is the vector of the stochastic disturbance terms in the reduced form. The corresponding deterministic reduced form of the model is (2.4). From (4.2) it follows that the econometric model uniquely specifies not the endogenous variables but rather the probability distribution of each of the endogenous variables, given the values taken by all exogenous variables and given the values of all parameters of the model. Each equation of the model, other than definitions, equilibrium conditions, and identities, is generally assumed to contain an additive stochastic disturbance term, which is an unobservable random variable with certain assumed properties, e.g. mean, variance, and covariance. The values taken by that variable are not known with certainty; rather, they can be considered random drawings from a probability distribution with certain assumed moments. The inclusion of such stochastic disturbance terms in the econometric model is basic to the use of tools of statistical inference to estimate parameters of the model.

Econometric models are either linear or non-linear. Early econometric models and many current econometric models are linear in that they can be expressed as models that are linear in the parameters. This linearity assumption has been an important one for proving mathematical and statistical theorems concerning econometric models, for estimating parameters, and for using the estimated models for structural analysis, forecasting, and policy evaluation. The linearity assumption has been justified in several ways. First, many economic relationships are by their very nature linear, such as the definitions of expenditure, revenue, cost, and profit. Second, the linearity assumption applies only to parameters, not to variables of the model. Thus, a quadratic cost function, of the form

$$C=a+bq+cq^2, \tag{4.3}$$

where C is cost, q is output, and a, b, and c are parameters, while non-linear in q, is linear in a, b, and c. Third, non-linear models can sometimes be transformed into linear models, such as by a logarithmic transformation. For example, the Cobb–Douglas production function

$$Y=AK^{\alpha}L^{\beta}, \tag{4.4}$$

where Y is output, K is capital, L is labor, and A, a, and β are parameters, can be so transformed into the log-linear form

$$\log Y = a + \alpha \log K + \beta \log L \qquad (a = \log A). \tag{4.5}$$

Fourth, any smooth function can be reasonably approximated in an appropriate range by a linear function, e.g. via a Taylor's theorem approximation. Consider, for example, the general production function

$$Y = F(K, L), \tag{4.6}$$

of which the Cobb–Douglas form (4.4) is one special case. If the function is continuous it can be approximated as a linear function in an appropriate range by taking the linear portion of the Taylor's series expansion. Expanding about the base levels of (K_0, L_0),

$$Y \cong F(K_0, L_0) + \frac{\partial F}{\partial K}(K_0, L_0)(K - K_0) + \frac{\partial F}{\partial L}(K_0, L_0)(L - L_0), \tag{4.7}$$

so that[8]

$$Y \cong a + bK + cL, \tag{4.8}$$

where the parameters can be interpreted as

$$\begin{aligned} a &= F(K_0, L_0) - \frac{\partial F}{\partial K}(K_0, L_0)K_0 - \frac{\partial F}{\partial L}(K_0, L_0)L_0, \\ b &= \frac{\partial F}{\partial K}(K_0, L_0), \\ c &= \frac{\partial F}{\partial L}(K_0, L_0). \end{aligned} \tag{4.9}$$

Finally, linear models are much more convenient and more manageable than

[8] Other approximations are also possible, e.g. expressing the production function as

$$\log Y = \phi(\log K, \log L).$$

Taking a Taylor's series approximation yields

$$\log Y \cong a' + b' \log K + c' \log L,$$

which would approximate any production function as a log-linear Cobb–Douglas production function as in (4.4) and (4.5). See Kmenta (1967). For a more general discussion of transformations see Box and Cox (1964).

non-linear models. Thus, the linearity assumption has frequently been made for econometric models.

Non-linear models, that is, econometric models that are non-linear in the parameters, have become more common in recent years largely due to advances in computer software and numerical analysis that have facilitated the estimation of such models. Techniques and computer software used in the estimation of non-linear econometric models are discussed in Chapters 6 and 12 of this Handbook by Takeshi Amemiya and Richard Quandt, respectively. The parameters of a non-linear model are frequently estimated using successive linear approximations to the model, and the properties of such estimators can be derived asymptotically or approximately. While these properties are valid for large samples the exact small sample properties of estimators for general non-linear econometric models are unknown. Furthermore, some of the properties have been shown to hold only under the assumption of normally distributed stochastic disturbances, and the consequences of model misspecification are generally not known in the non-linear case. A considerable amount of work has been done in recent years on non-linear models, however, as discussed elsewhere in this Handbook.[9]

4.1. *Structural form*

The basic econometric model is the structural form, from which the reduced form and the final form can be obtained. The general *structural form* of the linear (in parameters) stochastic econometric model, assuming there are g endogenous variables, $y_1, y_2, \dots, y_g$, and k exogenous variables, $x_1, x_2, \dots, x_k$, can be written:

$$
\begin{aligned}
&y_1\gamma_{11} + y_2\gamma_{21} + \cdots + y_g\gamma_{g1} + x_1\beta_{11} + x_2\beta_{21} + \cdots + x_k\beta_{k1} = \varepsilon_1, \\
&y_1\gamma_{12} + y_2\gamma_{22} + \cdots + y_g\gamma_{g2} + x_1\beta_{12} + x_2\beta_{22} + \cdots + x_k\beta_{k2} = \varepsilon_2, \\
&\vdots \\
&y_1\gamma_{1g} + y_2\gamma_{2g} + \cdots + y_g\gamma_{gg} + x_1\beta_{1g} + x_2\beta_{2g} + \cdots + x_k\beta_{kg} = \varepsilon_g.
\end{aligned}
\tag{4.10}
$$

Here the γ's are the coefficients of the endogenous variables, the β's are the coefficients of the exogenous variables, and $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_g$ are g stochastic disturbance terms (random variables). This system of equations can be considered the linear and stochastic version of the system (2.1), where the parameters include not only the coefficients but also those parameters characterizing the stochastic

[9]For discussions of non-linear models see Chapters 6 and 12 of this Handbook by Amemiya and Quandt, respectively. See also Goldfeld and Quandt (1968, 1972), Chow (1973), Jorgenson and Laffont (1974), Goldfeld and Quandt (1976), Belsley (1979, 1980), Gallant and Jorgenson (1979), Fair and Parke (1980), and Gallant and Holly (1980).

disturbance terms. Intercept terms in the equations can be taken into account by specifying one of the exogenous variables, conventionally either the first x_1 or the last x_k, to be identically unity, in which case its coefficients become the intercepts.

Typically, each equation of the structural form (4.10) has an independent meaning and identity, reflecting a behavioral relation (such as a demand function or a consumption function), a technological relation (such as a production function), or some other specific relation suggested by theory for the system under study. Each equation, because it represents one aspect of the structure of the system, is called a *structural equation*, and the set of all structural equations is the *structural form*. Some equations may be deterministic, e.g. definitions, identities, and equilibrium conditions, and for these equations the stochastic disturbance terms are identically zero. In general, however, these equations can be eliminated, reducing both the number of equations and the number of endogenous variables.

The structural form can also be written in *summation notation*, as

$$\sum_{h=1}^{g} y_h\gamma_{hl} + \sum_{j=1}^{k} x_j\beta_{jl} = \varepsilon_l, \qquad l = 1,2,\dots,g, \tag{4.11}$$

where h is an index of the endogenous variable, l is an index of the equation, and j is an index of the exogenous variable. In *vector–matrix notation* the structural form is written:

$$y\Gamma + xB = \varepsilon, \tag{4.12}$$

which is the linear version of system (4.1), where the coefficient matrices are

$$\Gamma = (\gamma_{hl}) \tag{4.13}$$

and

$$B = (\beta_{jl}). \tag{4.14}$$

Γ is a $g \times g$ matrix of coefficients of endogenous variables, assumed non-singular, and B is a $k \times g$ matrix of coefficients of exogenous variables. Note that the lth columns of Γ and B contain all coefficients in the lth equation of the structural form for $l = 1,2,\dots,g$. The structural form in vector-matrix notation in (4.12) is the most convenient of the three ways of expressing the structural form, and it will be used in the remainder of this chapter.

There is a trivial indeterminacy in the structural equations in that multiplying all terms in any one of these equations by a non-zero constant does not change the equation. This indeterminacy is eliminated by choosing a *normalization rule*, which is a rule for selecting a particular numerical value for one of the non-zero coefficients in each question. A convenient normalization rule is that which sets all elements along the principal diagonal of the Γ matrix of coefficients of endogenous variables at -1:

$$\gamma_{hh} = -1, \qquad h = 1,2,\dots,g. \tag{4.15}$$

This normalization rule, obtained by dividing all coefficients of equation h by $-\gamma_{hh}$, yields the usual convention of being able to write each equation which specifies one endogenous variable as a function of other endogenous variables, exogenous variables, and a stochastic disturbance term, with a unique such endogenous variable for each equation. Other normalization rules can be used, however, typically involving setting the (non-zero) coefficient of one variable in each equation as 1 or -1 (by dividing by this coefficient or its negative).

Letting i be an index of the observation number, the structural form at the ith observation is

$$y_i\Gamma + x_iB = \varepsilon_i, \qquad i = 1,2,\dots,n. \tag{4.16}$$

Here y_i, x_i, and ε_i are, respectively, the vector of endogenous variables, the vector of exogenous variables, and the vector of stochastic disturbance terms at the ith observation, where i ranges over the sample from 1 to n, n being the sample size (the number of observations). Certain stochastic assumptions are typically made concerning the n stochastic disturbance vectors ε_i. First, they are assumed to have a zero mean:

$$E(\varepsilon_i) = 0, \qquad i = 1,2,\dots,n. \tag{4.17}$$

Second, the covariance matrix of ε_i is assumed to be the same at each observation:

$$\operatorname{cov}(\varepsilon_i) = E(\varepsilon_i'\varepsilon_i) = \Sigma, \qquad i = 1,2,\dots,n, \tag{4.18}$$

where Σ, the positive definite symmetric matrix of variances and covariances, is the same for each i. Third, the ε_i are assumed uncorrelated over the sample

$$E(\varepsilon_i'\varepsilon_j) = 0, \qquad i = 1,2,\dots,n; \quad j = 1,2,\dots,n; \quad i \neq j, \tag{4.19}$$

so that each stochastic disturbance term is uncorrelated with any stochastic disturbance term (including itself) at any other point in the sample. These assumptions are satisfied if, for example, the stochastic disturbance vectors ε_i are independently and identically distributed over the sample, with a zero mean vector and a constant covariance matrix Σ. Sometimes the further assumption of normality is also made, specifying that the ε_i are distributed independently and normally with zero mean vector and $g \times g$ positive definite symmetric covariance matrix Σ:

$$\varepsilon_i \sim N(0, \Sigma), \qquad i = 1,2,\dots,n. \tag{4.20}$$

Under these general assumptions (without necessarily assuming normality), while the stochastic disturbance terms are uncorrelated over the sample, they can, by (4.18), be correlated between equations. This latter phenomenon of correlation between stochastic disturbance terms in different equations (due to the fact that there is usually more than one endogenous variable in each equation) is an essential feature of the simultaneous-equation system econometric model and the principal reason why it must be estimated using simultaneous-equation rather than single-equation techniques, as discussed in Chapter 7 of this Handbook by Jerry Hausman.

4.2. *Reduced form*

The structural form (4.10) is a special case of the general system (2.1) (other than the addition of stochastic disturbance terms). The general system could be solved for the endogenous variables if condition (2.3) is met. In the case of the structural form (2.3) is the condition that the matrix Γ of coefficients of endogenous variables be non-singular, which is usually assumed. Then the structural form can be solved for the endogenous variables as explicit (linear, stochastic) functions of all exogenous variables and stochastic disturbance terms – the reduced form. Postmultiplying (4.12) by Γ^{-1} and solving for y yields

$$y = -xB\Gamma^{-1} + \varepsilon\Gamma^{-1}. \tag{4.21}$$

Introducing the $k \times g$ matrix of reduced-form coefficients Π and the $1 \times g$ reduced-form stochastic disturbance vector u, where

$$\Pi \equiv -B\Gamma^{-1}, \qquad u \equiv \varepsilon\Gamma^{-1}, \tag{4.22}$$

the *reduced form* is written

$$y = x\Pi + u. \tag{4.23}$$

This reduced form uniquely determines the probability distributions of the endogenous variables, given the exogenous variables, the coefficients, and the probability distributions of the stochastic disturbance terms.

The matrix of reduced-form coefficients represents the changes in endogenous variables as exogenous variables change:

$$\Pi=\frac{\partial y}{\partial x}, \quad \text{i.e. } \Pi_{jh}=\frac{\partial y_h}{\partial x_j}, \qquad j=1,2,\dots,k; \quad h=1,2,\dots,g. \tag{4.24}$$

Thus, the elements of the matrix of reduced-form coefficients represent the comparative statics results of the model, the jh element of Π measuring the change in the hth endogenous variable as the jth exogenous variable changes, all other predetermined variables and all stochastic disturbance terms being held constant. The estimation of these comparative statics results is an important aspect of structural analysis using the econometric model.

The stochastic assumptions made for the structural form have direct implications for the stochastic disturbance terms of the reduced form. If i is an index of the observation number, the reduced form at the ith observation is

$$y_i=x_i\Pi+u_i, \qquad i=1,2,\dots,n, \tag{4.25}$$

where Π is the same as in (4.22) and the reduced-form stochastic disturbance vector is

$$u_i\equiv\varepsilon_i\Gamma^{-1}. \tag{4.26}$$

This identity is used to obtain conditions on u_i from those assumed for ε_i. From (4.17):

$$E(u_i)=0, \qquad i=1,2,\dots,n. \tag{4.27}$$

From (4.18):

$$\operatorname{cov}(u_i)=E(u_i'u_i)=(\Gamma^{-1})'E(\varepsilon_i'\varepsilon_i)\Gamma^{-1}=(\Gamma^{-1})'\Sigma\Gamma^{-1}=\Omega, \qquad i=1,2,\dots,n, \tag{4.28}$$

where Ω is the covariance matrix of u_i, which, as is the case of the covariance matrix Σ of ε_i, is constant over the sample. The last equality in (4.28) implies that

$$\Sigma=\Gamma'\Omega\Gamma, \tag{4.29}$$

showing the relationship between the covariance matrix of the structural form Σ and that of the reduced form Ω. Furthermore, from (4.19),

$$E(u_i'u_j)=0, \qquad i=1,2,\dots,n; \quad j=1,2,\dots,n; \quad i\neq j, \tag{4.30}$$

so the u_i, just as the ε_i, are uncorrelated over the sample. If it is further assumed that the ε_i are independently and normally distributed, as in (4.20), then the u_i are also independently and normally distributed, with zero mean vector and $g \times g$ positive definite symmetric covariance matrix Ω:

$$u_i \sim N(0,\Omega), \qquad i=1,2,\dots,n, \tag{4.31}$$

where Ω is given in (4.28) as $(\Gamma^{-1})'\Sigma\Gamma^{-1}$.

Assumptions (4.27), (4.28), and (4.30) summarize the stochastic specification of the reduced-form equations. Under these assumptions the conditions of both the Gauss–Markov Theorem and the Least Squares Consistency Theorem are satisfied for the reduced-form equations, so the least squares estimators

$$\hat{\Pi} = (X'X)^{-1}X'Y, \tag{4.32}$$

where X is the $n \times k$ matrix of data on the k exogenous variables at the n observations and Y is the $n \times g$ matrix of data on the g endogenous variables at the n observations, are the unique best linear unbiased and consistent estimators of the reduced form. The covariance matrix can then be estimated as

$$\hat{\Omega} = \frac{1}{n-k}(Y - X\hat{\Pi})'(Y - X\hat{\Pi}) = \frac{1}{n-k}Y'\left[I - X(X'X)^{-1}X'\right]Y, \tag{4.33}$$

where $I - X(X'X)^{-1}X'$ is the fundamental idempotent matrix of least squares, as introduced in Chapter 1 of this Handbook by Henri Theil. This estimator of the covariance matrix is an unbiased and consistent estimator of Ω.

4.3. *Final form*

Econometric models are either static or dynamic. A *static model* involves no explicit dependence on time, so time is not essential in the model. (Simply adding time subscripts to variables does not convert a static model into a dynamic one.) A *dynamic model* is one in which time plays an essential role, typically by the inclusion of lagged variables or differences of variables over time. Thus, if any equation of the model is a difference equation, then the model is dynamic. (Time also plays an essential role if variables and their rates of change over time are included in the model, such as in a differential equation.)

If the econometric model is dynamic in including lagged endogenous variables, then it is possible to derive another form of the model, the final form.[10] The final

[10]See Theil and Boot (1962).

form expresses the current endogenous variables as functions of base values and all relevant current and lagged exogenous and stochastic disturbance terms. If the structural form involves only one lag, then it can be written[11]

$$y_t\Gamma+\left(y_{t-1}\vdots x_t\right)\left(\frac{B_1}{B_2}\right)=\varepsilon_t \tag{4.34}$$

or

$$y_t\Gamma+y_{t-1}B_1+x_tB_2=\varepsilon_t, \tag{4.35}$$

where y_{t-1} is a vector of lagged endogenous variables. The lagged endogenous and exogenous variables are grouped together as the predetermined variables of the system. The B matrix has then been partitioned to conform to the partitioning of the predetermined variables into lagged endogenous and current exogenous variables. The reduced form is then

$$y_t=y_{t-1}\Pi_1+x_t\Pi_2+u_t, \tag{4.36}$$

where

$$\Pi_1\equiv-B_1\Gamma^{-1};\qquad \Pi_2\equiv-B_2\Pi^{-1}, \tag{4.37}$$

$$u_t\equiv\varepsilon_t\Gamma^{-1}. \tag{4.38}$$

The final form is obtained by solving this reduced form, which is a difference equation in y_t, iteratively for y_0. In the first iteration

$$\begin{aligned} y_t&=(y_{t-2}\Pi_1+x_{t-1}\Pi_2+u_{t-1})\Pi_1+x_t\Pi_2+u_t\\ &=y_{t-2}\Pi_1^2+[x_t\Pi_2+x_{t-1}\Pi_2\Pi_1]+[u_t+u_{t-1}\Pi_1]. \end{aligned} \tag{4.39}$$

[11] The index has been changed from i in (4.16) to t in (4.34) to emphasize the facts that the variables depend on time and that the model includes lagged variables. It should also be noted that any finite number of lags in both endogenous and exogenous variables can be treated using the approach of this section. With lags up to those of order p, eq. (4.35) generalizes to

$$y_t\Gamma+\sum_{j=1}^{p}y_{t-j}B_{1j}+\sum_{k=0}^{p}x_{t-k}B_{2k}=\varepsilon_t.$$

An infinite number of lags, of the form (for a single endogenous variable)

$$y_t=\alpha+\sum_{k=0}^{\infty}\beta_kx_{t-k}+u_t,$$

is a *distributed lag model*, discussed in Chapters 17–20 of this Handbook by Granger and Watson; Hendry, Pagan and Sargan; Geweke; and Bergstrom, respectively. See also Griliches (1967), Sims (1974), and Dhrymes (1981).

Continuing the iteration back to the base period $t=0$ yields

$$y_t = y_0 \Pi_1^t + \sum_{j=0}^{t-1} x_{t-j} \Pi_2 \Pi_1^j + \sum_{j=0}^{t-1} u_{t-j} \Pi_1^j. \tag{4.40}$$

This is the *final form*, in which each of the endogenous variables is expressed as a function of base period values, current and lagged exogenous variables, and stochastic disturbance terms. The coefficient of the base period endogenous variable is Π_1^t, and the successive coefficients of the current and lagged exogenous variables

$$\Pi_2, \quad \Pi_2 \Pi_1, \quad \Pi_2 \Pi_1^2, \ldots, \tag{4.41}$$

indicate the influence of the current value of the endogenous variables of successively lagged values of the exogenous variables, starting from the current (non-lagged) values and given as

$$\frac{\partial y_t}{\partial x_{t-j}} = \Pi_2 \Pi_1^j, \quad j = 1, 2, \ldots, t-1. \tag{4.42}$$

The estimation of these successive coefficients, which can be interpreted as various multipliers, is an important aspect of structural analysis using the econometric model.

5. Identification

The problem of identification is an important issue in econometric model building.[12] Most approaches to the estimation of the structural form start from the estimation of the reduced-form equations, specifically the estimated matrix of reduced-form coefficients $\hat{\Pi}$ in (4.32) and the estimated covariance matrix $\hat{\Omega}$ in (4.33) and use these estimators to obtain estimates of the structural form parameters Γ, B, and Σ in (4.12) and (4.18).

The *problem of identification* is that of using estimates of reduced-form parameters Π and Ω to obtain estimates of structural-form parameters Γ, B, and Σ. Certain information is available from the relations between the structural form and reduced form. In particular, from (4.22) and (4.29) if $\hat{\Pi}$ and $\hat{\Omega}$ are estimates of Π and Ω, respectively, while if $\hat{\Gamma}$, $\hat{B}$, and $\hat{\Sigma}$ are estimates of Γ, B, and Σ,

[12]For an extensive discussion of identification see Chapter 4 of this Handbook by Hsiao. Basic references on identification include Fisher (1966), Rothenberg (1971, 1973), and Bowden (1973). See also Intriligator (1978, ch. 10).

respectively, the estimates must satisfy

$$\hat{B} = \hat{\Pi}\hat{\Gamma} \tag{5.1}$$

and

$$\hat{\Sigma} = \hat{\Gamma}'\hat{\Omega}\hat{\Gamma}. \tag{5.2}$$

These restrictions provide *a posteriori* information, since they follow the estimation of the reduced form. This information, however, is generally not adequate to determine the structural parameters. For example, if the structural form were postmultiplied by any non-singular matrix R:

$$y_i\Gamma R + x_i BR = \varepsilon_i R, \tag{5.3}$$

and this "bogus" system were normalized in the same way as the old one, where the bogus parameters are

$$\bar{\Gamma} = \Gamma R; \qquad \bar{B} = B\Gamma; \qquad \bar{\Sigma} = R'\Sigma R, \tag{5.4}$$

then the reduced form is

$$\bar{\Pi} = -\bar{B}\bar{\Gamma}^{-1} = -BRR^{-1}\Gamma^{-1} = -B\Gamma^{-1} = \Pi, \tag{5.5}$$

$$\bar{\Omega} = (\bar{\Gamma}^{-1})'\bar{\Sigma}\bar{\Gamma}^{-1} = (\Gamma^{-1})'\Sigma\Gamma^{-1} = \Omega. \tag{5.6}$$

Thus, the bogus system has the same reduced-form parameters as the true system. The true and bogus systems are *observationally equivalent* in yielding the same reduced form (more precisely, in implying the same likelihood function for the observed values of the endogenous variables, given the values of the predetermined variables). Thus, the *a posteriori* information in (5.1) and (5.2) cannot distinguish between Γ, B, and Σ, the true parameters, and $\bar{\Gamma}$, $\bar{B}$, and $\bar{\Sigma}$, the bogus parameters. To distinguish the true parameters it is necessary to supplement the *a posteriori* information by *a priori information*, restrictions on the structural parameters imposed prior to the estimation of the reduced form. These restrictions on the structural form, obtained from relevant theory or the results of other studies, have the effect of reducing the class of permissible matrices R in (5.3). If no such restrictions are imposed, or too few are imposed, the system is *not identified*, in which case additional *a priori* information must be imposed in order to identify the structural parameters Γ, B, and Σ. If enough *a priori* information is available, then the system is *identified* in that all structural parameters can be determined from the reduced-form parameters. A structural equation is *underidentified* if there is no way to determine its parameters from the reduced-form parameters. It is *just identified* (or *exactly identified*) if there is a unique way of estimating its parameters from the reduced-form parameters. It is *overidentified* if there is more than one way to calculate its parameters from the reduced-form parameters, leading to restrictions on the reduced-form parameters.

The *a priori* restrictions on the structural-form parameters Γ, B, and Σ usually involve one of three approaches. The first approach is that of zero or linear restrictions, equating some elements of the coefficient matrices *a priori* to zero or, more generally, imposing a set of linear restrictions. The second approach is that of restrictions on the covariance matrix Σ, e.g. via zero restrictions or relative sizes of variances or covariances. A third approach is some mixture of the first two, where certain restrictions, in the form of equalities or inequalities, are imposed on Γ, B, and Σ. An example is that of a *recursive system*, where Γ is a triangular matrix and Σ is a diagonal matrix. Such a system is always just identified, each equation being just identified.[13]

6. Some specific models

This section will present some specific models that have been used in econometrics. It emphasizes systems of equations, as opposed to single equation models.[14]

6.1. Demand models

One of the earliest and most important applications of econometric models is to the estimation of demand relationships.[15] In fact, pioneer empirical analyses of demand, starting in the nineteenth century with the work of Engel and continuing in the early twentieth century with the work of Schultz and Moore, led to later studies of general issues in econometrics.

A complete system of demand equations for n goods consists of the n demand equations:

$$x_j = x_j(p_1, p_2, \dots, p_n, I, u_j), \qquad j = 1, 2, \dots, n, \tag{6.1}$$

where x_j is the demand for good j by a single household or a group of households, p_j is the price of good j, I is income, which is the same as the expenditure on the n

[13] For a discussion of recursive systems see Wold (1954, 1960) and Wold (1968).

[14] For a more extensive discussion of various models and a discussion of single equation models see Intriligator (1978, esp. ch. 7, 8, 9, 12, and 13).

[15] For an extensive discussion of demand analysis see Chapter 30 of this Handbook by Deaton. Basic references for econometric studies of consumer demand include Brown and Deaton (1972), Powell (1974), Phlips (1974), Theil (1975/1976), and Barten (1977). See also Intriligator (1978, ch. 7).

goods, and u_j is the stochastic term in the jth demand equation. The n equations determine the quantity demanded of each good, which are the n endogenous variables, as functions of all prices and income, the $n+1$ exogenous variables, and stochastic terms, the latter accounting for omitted variables, misspecification of the equation, and errors in measuring variables. These n equations are the principal results of the theory of the consumer, and their estimation is important in quantifying demand for purposes of structural analysis, forecasting, and policy evaluation.

In order to estimate the system (6.1) it is necessary to specify a particular functional form for the general relationship indicated, and a variety of functional forms has been utilized. Only three functional forms will be considered here, however.

A functional form that has been widely used in demand (and other) studies is the *constant elasticity, log-linear specification*.[16] The n demand functions in (6.1) are specified as

$$x_j = A_j p_1^{\varepsilon_{j1}} p_2^{\varepsilon_{j2}} \dots p_n^{\varepsilon_{jn}} I^{\eta_j} e^{u_j}, \qquad j=1,2,\dots,n, \tag{6.2}$$

so, taking logarithms leads to the log-linear system:

$$\begin{aligned} \ln x_j &= a_j + \varepsilon_{j1} \ln p_1 + \varepsilon_{j2} \ln p_2 + \cdots \varepsilon_{jn} \ln p_n + \eta_j \ln I + u_j, \\ a_j &= \ln A_j, \quad j=1,2,\dots,n. \end{aligned} \tag{6.3}$$

This system is one of constant elasticity, where ε_{jj} are the (own) price elasticities of demand:

$$\varepsilon_{jj} = \frac{\partial \ln x_j}{\partial \ln p_j} = \frac{p_j}{x_j} \frac{\partial x_j}{\partial p_j}, \qquad j=1,2,\dots,n, \tag{6.4}$$

the ε_{jk} for $j \neq k$ are the cross price elasticities of demand:

$$\varepsilon_{jk} = \frac{\partial \ln x_j}{\partial \ln p_k} = \frac{p_k}{x_j} \frac{\partial x_j}{\partial p_k}, \qquad j=1,2,\dots,n; \quad k=1,2,\dots,n, \tag{6.5}$$

[16]Among the studies using the constant elasticity log-linear specification are Wold and Jureen (1953), Stone (1954), and Houthakker (1957, 1965). While this is a frequently used specification of a system of demand equations, such a system is not consistent with the budget constraint and the theoretical restrictions on systems of demand equations discussed in Phlips (1974), Intriligator (1978), and Chapter 1 of this Handbook by Theil. At best it can be treated as a local approximation to the true system of demand equations.

and the η_j are the income elasticities of demand:

$$\eta_j = \frac{\partial \ln x_j}{\partial \ln I} = \frac{I}{x_j}\frac{\partial x_j}{\partial I}, \qquad j = 1,2,\ldots,n. \tag{6.6}$$

The defining characteristic of this specification is that all $n(n+1)$ of these elasticities ($n+1$ for each of the n goods) are constant.

Another functional form used in demand analysis is the *semilogarithmic specification*:[17]

$$x_j = a_j + b_{j1}\ln p_1 + b_{j2}\ln p_2 + \cdots + b_{jn}\ln p_n + c_j \ln I + u_j, \quad j = 1,2,\ldots,n, \tag{6.7}$$

where the coefficients are

$$b_{jk} = \frac{\partial x_j}{\partial \ln p_k} = p_k \frac{\partial x_j}{\partial p_k}, \tag{6.8}$$

so b_{jk}/x_j is the (own or cross) price elasticity of demand.

A third functional form which is widely used in studies of demand is the *linear expenditure system*.[18] This system is

$$x_j = x_j^0 + \frac{\beta_j}{p_j}\left(I - \Sigma p_k x_k^0\right), \quad \text{with } x_j^0 \geqslant 0, \quad j = 1,2,\ldots,n, \tag{6.9}$$

or, in terms of expenditure,

$$p_j x_j = p_j x_j^0 + \beta_j\left(I - \Sigma p_k x_k^0\right), \quad \text{with } x_j^0 \geqslant 0, \quad j = 1,2,\ldots,n. \tag{6.10}$$

It can be interpreted as stating that expenditure on good j is composed of two components, the first being expenditure on a certain base amount x_j^0, which is the amount to which the consumer is committed, and the second being a fraction β_j of the so-called "supernumerary income", given as the income above the "subsistence income" $\Sigma p_k x_k^0$ needed to purchase base amounts of all goods. These two

[17]See Prais and Houthakker (1955). As in the case of the constant elasticity log-linear specification this semilogarithmic specification is not consistent with the theoretical restrictions on systems of demand equations.

[18]Among the many studies using the linear expenditure system are Stone, Brown and Rowe (1965), Pollak and Wales (1969), Stone (1972), Phlips (1974), Deaton (1975), and Barten (1977). The linear expenditure system, unlike the preceding ones, is consistent with the theoretical restrictions on systems of demand equations.

components correspond, respectively, to committed and discretionary expenditure on good j. The parameters that define the system are the n *base quantities*, $x_1^0, x_2^0, \ldots, x_n^0$, and the n *marginal budget shares*, $\beta_1, \beta_2, \ldots, \beta_n$.

6.2. *Production models*

A second important area of modeling is that of production functions.[19] While many studies treat only the production function itself, a complete system involves the production function and the first-order conditions for profit maximization (under competition) using this production function. Thus, the complete system consists of the $n+1$ equations:

$$\begin{aligned} &y = f(x_1, x_2, \ldots, x_n, u) \\ &\frac{1}{w_j}\frac{\partial f}{\partial x_j} = g(v_j), \qquad j = 1, 2, \ldots, n, \end{aligned} \tag{6.11}$$

where y is output, $x_1, x_2, \ldots, x_n$ are the n inputs, u is a stochastic disturbance term affecting technical efficiency, $f(\cdot)$ is the production function, w_j is the wage of input j relative to the price of output, $\partial f/\partial x_j$ is the marginal product of input j, v_j are stochastic disturbance terms affecting attainment of the first-order conditions, and $g(\cdot)$ is a function expressing how well the firm approximates the first-order conditions, under which g should be unity. These $n+1$ equations determine the output and the n inputs (the endogenous variables), as functions of the wages which, assuming the firm takes prices as given, are the exogenous variables. Estimation of this complete system is generally superior to estimating only the first equation from both an economic and an econometric standpoint. From an economic standpoint, estimating only the first equation reflects only the technology available to the firm, while estimating the complete system reflects the behavior of the firm (profit-maximizing) as well as the technology available to it. From an econometric standpoint estimating the first equation involves simultaneous equations bias, while estimating the complete system can result in consistent estimators. Even estimating the complete system cannot, however, be used to test the hypothesis of profit maximization, which is assumed in (6.11). Furthermore, the system (6.11) assumes that the correct prices and decision rules are known. If they are not known, then the system involves unknown parameters or functional forms, while if they are incorrect, then specification error is introduced in the system.

[19] For an extensive discussion of production (and cost) analysis see Chapter 31 of this Handbook by Jorgenson. Basic references for econometric studies of production functions include Walters (1963, 1968), Frisch (1965), Brown (1967), and Ferguson (1969). See also Intriligator (1978, ch. 8).

As in the case of demand analysis, a variety of functional forms has been utilized in estimating (6.11), but only three will be considered here.

One of the most widely used functional forms for production functions is the same as that used in demand analysis, *the constant elasticity log-linear specification*, also known as the *Cobb–Douglas production function*.[20] This function, already introduced in eqs. (4.4) and (4.5), can be written generally in the form of (6.11) as

$$\begin{aligned} &y = Ax_1^{\alpha_1}x_2^{\alpha_2}\dots x_n^{\alpha_n}e^u, \\ &\frac{1}{w_j}\frac{\partial y}{\partial x_j} = \frac{\alpha_j y}{w_j x_j} = e^{v_j}, \qquad j = 1,2,\dots,n, \end{aligned} \tag{6.12}$$

where disturbances are treated as exponential. Taking logarithms gives the linear system:

$$\begin{aligned} &\ln y = a + \alpha_1 \ln x_1 + \alpha_2 \ln x_2 + \cdots + \alpha_n \ln x_n + u, \qquad && a = \ln A, \\ &\ln y = \ln w_j + \ln x_j - \ln \alpha_j + v_j, && j = 1,2,\dots,n. \end{aligned} \tag{6.13}$$

A second widely used specification is the *constant elasticity of substitution* (*CES*) *production function*.[21] In the customary two-input case this function is

$$y = A\left[\delta x_1^{-\beta} + (1-\delta)x_2^{-\beta}\right]^{-1/\beta}, \tag{6.14}$$

where $\beta \geqslant -1$, the substitution parameter, is related to the elasticity of substitution σ by

$$\sigma = \frac{1}{1+\beta}. \tag{6.15}$$

This function reduces to the Cobb–Douglas case as $\beta \to 0$ (so $\sigma \to 1$); it reduces to a linear function as $\beta \to -1$ (so $\sigma \to \infty$); and it reduces to the input–output case of fixed coefficients as $\beta \to \infty$ (so $\sigma \to 0$).

A third specification is the *transcendental logarithmic* (*translog*) *production function*[22]

$$\ln y = a + \sum_{j=1}^{n} \alpha_j \ln x_j + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln x_i \ln x_j, \tag{6.16}$$

[20]See Marschak and Andrews (1944), Douglas (1948), Nerlove (1965), and Zellner, Kmenta and Drèze (1966).

[21]See Arrow, Chenery, Minhas and Solow (1961), Brown and de Cani (1963), and Minhas (1963).

[22]See Christensen, Jorgenson and Lau (1973).

where $\gamma_{ij}=\gamma_{ji}$. This function, which is quadratic in the logarithms of the variables, reduces to the Cobb–Douglas case if $\gamma_{ij}=0$; otherwise it exhibits non-unitary elasticity of substitution. In general this function is quite flexible in approximating arbitrary production technologies, providing a local approximation to any production frontier. It has also been applied to other frontiers, e.g. to demand functions or to price frontiers.

6.3. *Macroeconometric models*

Macroeconometric models, starting with the work of Tinbergen in the 1930s, represent one of the most important applications of econometrics.[23] Such models generally utilize a Keynesian framework for the determination of national income [usually measured as Gross National Product (GNP) or Gross Domestic Product (GDP)] and its components, consumption, investment, government, and net foreign investment, as well as other macroeconomic variables, such as the distribution of income, prices, wages, interest rates, employment, unemployment, production, and assets. These models are used for all three purposes of econometrics: structural analysis, forecasting, and policy evaluation.

Most macroeconometric models are built around the definition of income, a consumption function, and an investment function:

$$\begin{aligned} &Y=C+I+G,\\ &C=C(Y,\dots,u),\\ &I=I(Y,\dots,v). \end{aligned} \tag{6.17}$$

[23]For further discussions of macroeconometric models see Chapters 33, 34, and 35 of this Handbook, by Fair, Taylor, and Klein, respectively. For surveys of macroeconometric models see Nerlove (1966), Ball (1973), Fromm and Klein (1973), Samuelson (1975), and Klein and Burmeister (1976). For references to econometric models of the United States see Intriligator (1978, footnotes 24–28 and table 12.12 on pp. 454–456). Econometric models have also been developed for other national economics. See the references in Shapiro and Halabuk (1976) and Intriligator (1978). Some examples are Klein et al. (1961), Ball and Burns (1968), Hilton and Heathfield (1970), Ball (1973), Hendry (1974), and Renton (1975) for models of the United Kingdom; Brown (1960) and Helliwell et al. (1969, 1971) for models of Canada; Klein and Shinkai (1963), Ueno (1963), Ichimura et al. (1964), and Kosobud and Minami (1977) for models of Japan; Suits (1964) for a model of Greece; Evans (1969b) for a model of France; Evans (1970) for a model of Israel; Agarwala (1970) for a model of India; and Sylos-Labini (1974) for a model of Italy. In several of these countries these models have been further developed by official agencies, such as the Treasury model of the United Kingdom, the Bank of Japan and Economic Planning Agency models of Japan, and the Bundesbank model for the Federal Republic of Germany. Econometric models have also been built for most centrally planned economies, including the U.S.S.R., the German Democratic Republic, Czechoslovakia, Hungary, and the People's Republic of China. An interesting feature of some of the latter models is the mixture of deterministic and stochastic mechanisms for resource allocation, allowing some discretion for both planners' behavior and market-type mechanisms.

Here Y, national income, is composed of consumption, C, investment, I, and government expenditure, G. Consumption is determined by the consumption function $C(\cdot)$ as a function of income, other relevant variables (e.g. permanent income, assets, measures of income distribution) and a stochastic disturbance term, u. Investment is determined by the investment function $I(\cdot)$ as a function of income, other relevant variables (e.g. lagged income, profits, interest rates), and a stochastic disturbance term, v. Virtually all macroeconometric models involve these basic elements: a definition of national income or a group of such definitions, a consumption function or a group of such functions, and an investment function or a group of such functions. They have, in recent years, however, involved a greater and greater degree of disaggregation of variables and more and more variables as more aspects of the macroeconomy are taken into account.

The early postwar models involved less than ten stochastic equations, an example being the *Klein interwar model* of the U.S. economy over the period 1921–1941, involving three stochastic and three non-stochastic equations in six endogenous and four exogenous variables.[24]

An extremely influential model of the mid-1950s was the *Klein–Goldberger model* of the U.S. economy over the periods 1929–1941 and 1946–1952, which involved 15 stochastic and 5 non-stochastic equations in 20 endogenous and 14 exogenous variables.[25] Among the descendent econometric models of the U.S. economy based in part on or influenced by the Klein–Goldberger model are two models developed in the late 1960s and early 1970s, the Brookings model and the Wharton model.

The Brookings model was, at the time of its development in the 1960s the largest and most ambitious model of the U.S. economy, involving in its "standard" version 176 endogenous and 89 exogenous variables.[26] A major goal in building this model was that of advancing the state of the art in model building both via disaggregation and via the inclusion of sectors not treated in previous models. The resulting model, in representing the detailed structure of the economy, has been used both for structural analysis of cycles and for growth and policy evaluation.

The Wharton model was initiated in 1967 with a quarterly model of 76 endogenous and 42 exogenous variables. Since then later variants have involved the 1972 Wharton Annual and Industry model, with 346 endogenous and 90

[24]See Klein (1950) and Theil and Boot (1962). This model is also discussed in Christ (1966) and Theil (1971).

[25]See Klein and Goldberger (1955), Goldberger (1959), and Adelman and Adelman (1959).

[26]For the Brookings model see Duesenberry, Fromm, Klein and Kuh (1965, 1969), Fromm and Taubman (1968), Fromm (1971), and Fromm and Klein (1975).

exogenous variables, and the 1972 Wharton, Mark III model, with 201 endogenous and 104 exogenous variables.[27] The Wharton models are designed explicitly for developing forecasts of the future of the economy, particularly national income components and unemployment. They are regularly used to forecast ahead eight or more quarters under alternative assumptions regarding the exogenous variables, particularly government monetary and fiscal policy.

More recent macroeconometric models of the U.S., which stem largely from the Brookings and the Wharton models, include the 1968 FMP/MPS model of the Federal Reserve Board/MIT-Penn-SSRC, with 171 endogenous and 119 exogenous variables which emphasizes the monetary and financial sectors; the 1971 Chase Econometrics model, with 150 endogenous and 100 exogenous variables; the 1971 Fair short-run forecasting model with 19 endogenous and 20 exogenous variables; and the 1974 Data Resources Incorporated (DRI) model, with 718 endogenous and 170 exogenous variables in its initial version.[28]

Macroeconometric models have clearly tended to increase in size and scope, involving more variables, more sectors, and the inclusion of related models, such as input–output, financial, and microsimulation models. They have also tended to increase in complexity, including non-linearities. Another trend is the attempt to link various national macroeconometric models into a model of world trade flows.[29] These trends can be understood in terms of a rational response to the falling cost of scale and complexity, given the falling cost of computation and the availability of library programs for econometric routines and computerized data banks. These trends can be expected to continue in the future.

6.4. *Other econometric models*

An important trend in econometric models, particularly in macroeconometric models, has been that of the growing size, scale, and complexity of these models, as noted in the previous section. Another trend in econometric models has been that of applying such models in many other areas of economics, including fields in which such models have traditionally not been applied. The growing knowledge of econometric approaches and availability of library programs for economic

[27]For the Wharton model see Evans and Klein (1967, 1968), Evans (1969a), Evans, Klein and Saito (1972), and Howrey (1972). For the Wharton Annual and Industry model see Preston (1972, 1975). For the Wharton Mark III Quarterly model see McCarthy (1972), Duggal, Klein and McCarthy (1974), and Klein and Young (1980).

[28]For the FMP/MPS model see Rasche and Shapiro (1968), de Leeuw and Gramlich (1968, 1969), Ando and Modigliani (1969), Ando, Modigliani and Rasche (1972), and Muench et al. (1974). For the Chase Econometrics model see Evans (1974). For the Fair model see Fair (1971, 1974, 1976, 1979). For the DRI model see Eckstein, Green and Sinai (1974), Eckstein (1976), and Data Resources, Inc. (1976).

[29]Project LINK represents such an approach. See Ball (1973).

routines and data, including computerized data banks, have aided or accelerated this trend. In fact, econometric models have been developed in virtually all areas of economics, including commodity markets, crime, education, economic history, education, energy, health economics, housing, industrial organization, inflation, international trade and finance, labor economics, monetary economics, transportation, and urban and regional economics. In addition, there have been applications of econometric models to other social sciences, including demography, political science, and sociology.[30]

7. Uses of econometric models

The three principal uses of econometric models are structural analysis, forecasting, and policy evaluation, corresponding to the descriptive, predictive, and prescriptive uses of econometrics. These uses are closely related, the structure determined by structural analysis being used in forecasting, and policy evaluation being based largely on conditional forecasts. These uses will be discussed for both the general non-linear econometric model in (4.1) and (4.2) and the linear econometric model in (4.12) and (4.23).

Writing these models in vector notation and showing the lagged endogenous variables explicitly, the structural form of the general non-linear econometric model (4.1) can be written

$$f(y_t, y_{t-1}, x_t, \delta) = \varepsilon_t, \tag{7.1}$$

where y_t and y_{t-1} are vectors of current and lagged endogenous variables, x_t is a vector of exogenous variables at time t, δ is a vector of parameters, and ε_t is a vector of stochastic disturbance terms at time t. The corresponding reduced form of the general non-linear econometric model in (4.2) is

$$y_t = \phi(y_{t-1}, x_t, \delta) + u_t, \tag{7.2}$$

where u_t is a vector of stochastic disturbance terms for the reduced form at time t. The structural form of the general linear econometric model, also allowing for lagged endogenous variables, is

$$y_t\Gamma + y_{t-1}B_1 + x_tB_2 = \varepsilon_t, \tag{7.3}$$

[30]For discussions of and references to the applications of econometric models in these areas see Intriligator (1978, ch. 9, 13). See especially the bibliography for Chapter 9, which includes references to a wide range of applications of econometrics.

as in (4.35), while the corresponding reduced form of the general linear econometric model is

$$y_t = y_{t-1}\Pi_1 + x_t\Pi_2 + u_t, \tag{7.4}$$

as in (4.36).

7.1. Structural analysis

Structural analysis refers to the use of an estimated econometric model for the quantitative measurement of the underlying interrelationships of the system under consideration. One aspect of structural analysis is the estimation of the parameters of the structural form, particularly the elements of δ in (7.1) and the elements of the Γ, B_1, and B_2 matrices of structural coefficients in (7.3), in order to measure the extent of influence of each of the included variables in any equation of the model.

Another aspect of structural analysis is the estimation of the parameters of the reduced form, particularly the elements of the Π_1 and Π_2 matrices of reduced-form coefficients in (7.4). As indicated in (4.24), these coefficients have the interpretation of comparative statics results of the model, the effects of each of the exogenous variables on each of the endogenous variables of the model. These coefficients are also called *impact multipliers* since they indicate the impact of a change in a current value of an exogenous variable on the current value of an endogenous variable.

A third aspect of structural analysis, for a model with lagged endogenous variables, is the estimation of the final form (4.40), particularly the successive coefficients of the current and lagged exogenous variables in (4.41). These coefficients can also be used to estimate the interim and long-term multipliers of the econometric model. The *τ-period cumulative multiplier* measures the effect on each of the endogenous variables of a change in each of the exogenous variables over τ periods, given as

$$\left.\frac{\partial y}{\partial x}\right|_{\tau} = \sum_{j=0}^{\tau-1} \Pi_2\Pi_1^j = \Pi_2\left(I + \Pi_1 + \Pi_1^2 + \cdots + \Pi_1^{\tau-1}\right), \tag{7.5}$$

where I is the identity matrix.

Setting $\tau = 1$ yields the impact multipliers Π_2, the coefficients of the exogenous variables in the reduced form (4.36). Finite values of τ larger than 1 yield the *cumulative interim multipliers*, indicating the change in each endogenous variable as each exogenous variable experiences a sustained increase over τ periods.

Taking the limit as $\tau \to \infty$ yields the *long-term multipliers*:

$$\left.\frac{\partial y}{\partial x}\right|_{\infty} = \lim_{\tau \to \infty} \Pi_2\left(I + \Pi_1 + \Pi_1^2 + \cdots + \Pi_1^{\tau}\right) = \Pi_2(I - \Pi_1)^{-1}, \tag{7.6}$$

assuming the power series converges.[31] These long-term multipliers measure the effect on the endogenous variables of a permanent sustained increase in the exogenous variables.

7.2. *Forecasting*

Forecasting refers to the use of an estimated econometric model to predict quantitative values for certain variables, typically the endogenous variables of the model, outside the sample of data actually observed – typically a prediction for other times or places.[32]

The econometric approach to forecasting is typically based on the reduced form system, which, for the general non-linear case, can be written as in (7.2). A *short-run ex-ante forecast* of values taken by the endogenous variables at time $T+1$, given their values at time T, is then

$$\hat{y}_{T+1} = \phi\left(y_T, \hat{x}_{T+1}, \hat{\delta}\right) + \hat{u}_{T+1}. \tag{7.7}$$

The left-hand side, $\hat{y}_{T+1}$, represents the values forecasted for the endogenous variables at time $T+1$. Three sets of variables enter the $\phi(\cdot)$ function in (7.7). The first set of variables is y_T, the (current) values of the endogenous variables at

[31] The power series in (7.2), called a *Neumann expansion*, converges if $\lim \Pi_1^{\tau} = 0$, or, equivalently, if all characteristic roots of Π_1 have modulus less than unity. The long-term multiplier in (7.2) could have been obtained directly from (4.36) by noting that, in the long run, $y_t = y_{t-1}$, so

$$y_t = y_t\Pi_1 + x_t\Pi_2 + u_t.$$

Solving for y_t:

$$y_t = x_t\Pi_2(I - \Pi_1)^{-1} + u_t(I - \Pi_1)^{-1},$$

implying (7.6).

[32] For a discussion of forecasting using an econometric model see Chapter 33 of this Handbook by Fair. Basic references on econometric forecasting are Theil (1961, 1966), Zarnowitz (1967), Klein (1971), and Christ (1975). See also Intriligator (1980, ch. 15). For discussions of forecasting based on autoregressive-moving average (ARMA) models see Box and Jenkins (1970), Cooper (1972), Zellner and Palm (1974), Aitchison and Dunsmore (1975), Nicholls et al. (1975), Pindyck and Rubinfeld (1976), Granger and Newbold (1977), Palm (1977), and Nerlove, Grether and Carvalho (1979).

time T, summarizing the systematic dependence on lagged values of the endogenous variables due to constant growth processes, distributed lag phenomena, etc. The second set of variables in $\phi(\cdot)$ in (7.7) is $\hat{x}_{T+1}$, the predicted future values of the exogenous variables. Since the x's are exogenous and hence determined on the basis of factors not explicitly treated in the econometric model, it is reasonable to require that these variables be forecast on the basis of factors other than those treated in the model itself, such as on the basis of extrapolation of past trends, expert opinion, or forecasts from another econometric model (as one example, an econometric model for an industry might use forecasts from a macroeconometric model). The third set of variables in $\phi(\cdot)$ in (7.7) is $\hat{\delta}$, representing the estimated parameters of the econometric model. The final term in (7.7) is $\hat{u}_{T+1}$, the "add factors", which can be interpreted as estimates of future values of the disturbance terms (or, alternatively, as adjustments of intercepts in each of the reduced-form equations). These add factors account for omitted variables, incorrect specification, and errors in measuring variables, which were the reasons for including the stochastic disturbance term in the first place. Their inclusion in short-term *ex-ante* forecasts is appropriate; excluding such terms would be tantamount to ignoring relevant considerations simply because they were omitted from the model. For example, in macroeconometric model forecasting it would be inappropriate to ignore major strikes, external shocks, or new technologies simply because they were not explicitly included in the model. Of course, the add factors are subjective, varying from individual to individual and thus not replicable. Their inclusion thus means that subjective expert opinion is combined with objective factors in generating forecasts. Experience indicates that such judgmental add factors can improve significantly on the accuracy of forecasts made with an econometric model.[33]

In the case of a linear econometric model the short-term *ex-ante* forecast, based on the reduced-form equation (7.4), takes the form

$$\hat{y}_{T+1} = y_T\hat{\Pi}_1 + \hat{x}_{T+1}\hat{\Pi}_2 + \hat{u}_{T+1}. \tag{7.8}$$

[33]The choice of values for the add factors $\hat{u}_{T+1}$ can also be guided by past residuals in estimating the model and past forecast errors, which provide clues to omitted variables, errors in measuring coefficients, and systematic biases in forecasting exogenous variables. For example, one approach is to define add factors so that the computed values of the endogenous variables at the most recent observation, as adjusted by the add factors, are the same as the observed values. See Klein (1971) and Haitovsky, Treyz and Su (1974). It should be noted that professional opinion is by no means unanimous on the subject of add factors. While most of the commercially available macroeconometric models use add factors, Ray Fair has argued against their use. Fair argues, as in Chapter 33 of the Handbook, that the use of add factors means that the information contained in *ex-ante* forecasts has no scientific value. From this point of view he argues that the inclusion of add factors is not appropriate. Others use add factors freely in actual forecasts. A reasonable intermediate position would be to assert that add factors are appropriate in attempting to obtain the most accurate *ex-ante* forecast for a given period, but not appropriate for using predictions from a model for testing and comparison purposes.

Here y_T, $\hat{x}_{T+1}$, and $\hat{u}_{T+1}$ are vectors as before, and $\hat{\Pi}_1$ and $\hat{\Pi}_2$ are matrices of estimated coefficients, summarizing the partial dependence of the forecasted values for the endogenous variables at time $T+1$ on their values at time T and on the values of the forecasted values of the exogenous variables at time $T+1$, respectively. The terms on the right-hand side of (7.8) show explicitly the (linear) dependence on current values of the endogenous variables, on future values of the exogenous variables, and on add factors, representing a combination of objective factors, in $y_T\hat{\Pi}_1$ and $\hat{x}_{T+1}\hat{\Pi}_2$, and subjective factors, in $\hat{u}_{T+1}$.

The econometric forecasts in (7.7) and (7.8) are called "*ex-ante* forecasts" because they are true forecasts, made before the event occurs. By contrast, an *ex-post* forecast, made after the event, would replace predicted values of the exogenous variables by their actual values and would replace the add factors by the zero expected values of the stochastic disturbance terms. Thus, the *short-term ex-post forecast* is, for the non-linear model:

$$\hat{\hat{y}}_{T+1} = \phi(y_T, x_{T+1}, \hat{\delta}) \tag{7.9}$$

and, for the linear model:

$$\hat{\hat{y}}_{T+1} = y_T\hat{\Pi}_1 + x_{T+1}\hat{\Pi}_2 = \hat{y}_{T+1} + (x_{T+1} - \hat{x}_{T+1})\hat{\Pi}_2 - \hat{u}_{T+1}. \tag{7.10}$$

This *ex-post* forecast is useful in focusing on the explicitly estimated parts of the forecast, particularly the estimated coefficient matrices $\hat{\Pi}_1$ and $\hat{\Pi}_2$, eliminating the influence of $\hat{x}_{T+1}$ and $\hat{u}_{T+1}$, which are generally not explicitly estimated. It is possible to replicate *ex-post* forecasts, but not *ex-ante* forecasts. Furthermore, this forecast is optimal given a quadratic loss function.

There are several advantages to the econometric approach to forecasting in (7.7)–(7.10).[34] First, it provides a useful structure in which to consider explicitly various factors, including past values of variables to be forecast, values of other variables, and judgmental factors. Second, it leads to forecasts of variables that are consistent with one another since they must satisfy the requirements of the model, particularly its identities. Third, it leads to forecasts that are explicitly

[34] Both (7.4) and (7.5) refer to short-term forecasts. Long-term forecasts over a forecasting horizon h determine $\hat{y}_{T+h}$ on the basis of a succession of short-term forecasts or, equivalently, on the basis of the final form in (4.40), where the forecast of the endogenous variables at time $T+h$ is

$$\hat{y}_{T+h} = y_T\hat{\Pi}_1^h + \sum_{j=0}^{h-1} \hat{x}_{T+h-j}\hat{\Pi}_2\hat{\Pi}_1^j + \sum_{j=0}^{h-1} \hat{u}_{T+h-j}\hat{\Pi}_1^j.$$

Here y_T is the current value, as in (7.8), the $\hat{x}_{T+h-j}$ are successive expected future values of exogenous variables, and the $\hat{u}_{T+h-j}$ are successive expected future values of stochastic disturbance terms, where the last term on the right can itself be interpreted as the add factor for the long-term forecast.

conditional on current values of endogenous variables, expected future values of exogenous variables, add factors, and estimated coefficient matrices, facilitating analysis of the relative importance of each of these factors and tests of sensitivity. Fourth, and perhaps most important, it has a good record for accuracy and usefulness as compared to other approaches which tend to emphasize one aspect of an econometric forecast but exclude other aspects.

7.3. *Policy evaluation*

Policy evaluation refers to the use of an estimated econometric model to choose among alternative policies.[35] Assume there is a set of policy variables included among the exogenous variables of the model. The structural form (4.35) can then be written, for the non-linear model:

$$f(y_t, y_{t-1}, z_t, r_t, \delta) = \varepsilon_t, \tag{7.11}$$

and, for the linear model:

$$y_t\Gamma + y_{t-1}B_1 + z_tB_2 + r_tB_3 = \varepsilon_t, \tag{7.12}$$

where the vector x_t of exogenous variables has been divided into a vector of (non-policy) exogenous variables z_t and a vector of policy variables r_t, called the *instruments*. The corresponding reduced form is, for the non-linear model:

$$y_t = \phi(y_{t-1}, z_t, r_t, \delta) + u_t, \tag{7.13}$$

and, for the linear model:

$$y_t = y_{t-1}\Pi_1 + z_t\Pi_2 + r_t\Pi_3 + u_t, \tag{7.14}$$

where, in addition to (4.37) and (4.38),

$$\Pi_3 \equiv -B_3\Gamma^{-1}. \tag{7.15}$$

The problem of short-term policy evaluation is that of choosing at time T a particular set of policy variables for time $T+1$, given as r^*_{T+1}, where it is assumed that y_T is known. There are at least three alternative approaches to evaluating

[35]For further discussions of policy evaluation using an econometric model, see Chapters 34 and 35 of this Handbook by Taylor and Klein, respectively. Basic references on econometric policy evaluation are Tinbergen (1955, 1956), Theil (1961, 1964), Suits (1962), Hickman (1965), Fox, Sengupta and Thorbecke (1966), Naylor, Wertz and Wonnacott (1968), Naylor (1971), and Klein (1971, 1977). See also Intriligator (1978, ch. 16).

policy: the *instruments–targets approach*, the *social-welfare-function approach*, and the *simulation approach*.

In the *instruments–targets approach* it is assumed that there is a target for the endogenous variables y^*_{T+1}. The optimal instruments for the non-linear econometric model then solve the equation

$$y^*_{T+1} = \phi\left(y_T, \hat{z}_{T+1}, r^*_{T+1}, \hat{\delta}\right) + \hat{u}_{T+1}, \tag{7.16}$$

for r^*_{T+1}, where $\hat{z}_{T+1}$ is the vector of expected future values of the exogenous variables, $\hat{\delta}$ is the vector of estimated parameters, and $\hat{u}_{T+1}$ is a vector of add factors. In the linear case it is usually assumed that the number of instruments equals the number of targets, g, so the B_3 matrix is square, as is Π_3.[36] Assuming Π_3 is non-singular, and solving (7.14) for r^*_{T+1} yields

$$r^*_{T+1} = y^*_{T+1}\hat{\Pi}_3^{-1} - y_T\hat{\Pi}_1\hat{\Pi}_3^{-1} - \hat{z}_{T+1}\hat{\Pi}_2\hat{\Pi}_3^{-1} - \hat{u}_{T+1}\hat{\Pi}_3^{-1}, \tag{7.17}$$

giving the optimal value for the instruments as linear functions of the targets, the current values of the endogenous variables, the expected future values of the exogenous variables, and the add factors. This equation indicates the basic interdependence of policies and objectives, with optimal values of each instrument in general depending on all target variables. This approach leads to specific results, but it suffers from three difficulties: it does not allow for tradeoffs among the targets, it assumes that policymakers can specify targets, and it assumes there are enough independent instruments available.

The *social-welfare-function approach* to policy evaluation allows tradeoffs among the endogenous variables by assuming the existence of a social welfare function to be maximized by choice of the instruments subject to the constraints of the model. If $W(y_{T+1}, r_{T+1})$ is the social welfare function, dependent on both endogenous variables and policy variables in the next period, the problem is

$$\max_{r_{T+1}} W(y_{T+1}, r_{T+1}) \tag{7.18}$$

subject to the constraints of the econometric model. In the case of the non-linear model, W is maximized subject to (7.13), so the problem becomes

$$\max_{r_{T+1}} W\left(\phi\left(y_T, \hat{z}_{T+1}, r_{T+1}, \hat{\delta}\right) + \hat{u}_{T+1}, r_{T+1}\right), \tag{7.19}$$

while in the case of the linear model, W is maximized subject to (7.14), so the

[36]More generally, the targets could be a subset of the endogenous variables, and the number of instruments can exceed (or equal) the number of targets, the difference between the number of instruments and the number of targets being the *policy degrees of freedom*.

problem becomes

$$\max_{r_{T+1}} W\left(y_T\hat{\Pi}_1 + \hat{z}_{T+1}\hat{\Pi}_2 + r_{T+1}\hat{\Pi}_3 + \hat{u}_{T+1}, r_{T+1}\right). \tag{7.20}$$

Frequently the social welfare function is a quadratic loss function, to be minimized. While this approach allows for tradeoffs among the endogenous variables it assumes that policymakers can specify a social welfare function.[37]

The *simulation approach* to policy evaluation does not require either targets or a social welfare function. This approach uses the estimated reduced form to determine alternative combinations of policy variables and endogenous variables for a given set of possible policies. If $r^1_{T+1}, r^2_{T+1}, \dots, r^S_{T+1}$ represent a set of alternative possible policies the simulation approach would determine the endogenous variables implied by each such policy, where, in the non-linear case,

$$\hat{y}^q_{T+1} = \phi\left(y_T, \hat{z}_{T+1}, r^q_{T+1}, \hat{\delta}\right) + \hat{u}_{T+1}, \tag{7.21}$$

and, in the linear case,

$$\hat{y}^q_{T+1} = y_T\hat{\Pi}_1 + \hat{z}_{T+1}\hat{\Pi}_2 + r^q_{T+1}\hat{\Pi}_3 + \hat{u}_{T+1}, \qquad q = 1, 2, \dots, S. \tag{7.22}$$

The policymaker would provide the model builder with the alternative policies, and the model builder would, in turn, provide the decisionmaker with their consequences for the endogenous variables. The policymaker would then choose a desired policy and its outcome, r^*_{T+1}, y^*_{T+1}, where r^*_{T+1} is one of the alternative policies available.[38] This approach does not require information on the tastes of the policymaker, such as targets or a social welfare function. Rather, it requires that the policymaker formulate an explicit set of policy alternatives and that an estimated econometric model incorporating the appropriate policy variables be available. Simulation, based in part on communication between policymaker and model builder, represents a valuable approach to policy evaluation that could be used in any policy area in which there exists a relevant estimated econometric model.

[37]See Pindyck (1973), Chow (1975, 1981), Ando and Palash (1976), Klein (1977), and Fair (1978) for an extension of this approach to the problem of optimal control using an econometric model, involving the choice of a time path for policy variables so as to maximize the sum over time (or, in the continuous case, the integral over time) of a social welfare function.

[38]There may, of course, be a problem of simultaneously making a consistent model of expectations of other agents and choosing optimal behavior, as is recognized in the rational expectations literature. In particular, when it becomes necessary to model how agents formulate expectations with respect to alternative policies, some of the structural parameters might themselves change as a result of policy choices, creating severe problems in estimating an econometric model. See Lucas and Sargent (1980) and Sargent (1981).

8. Conclusion

This survey of economic and econometric models indicates that there is a wide range of models and applications. There are many approaches to modeling, and even in the standard linear stochastic algebraic model of econometrics there are many alternative specifications available. These models have been applied in many different areas; in fact, in virtually all areas of economics and in some related social sciences. The models have been used for various purposes, including structural analysis, forecasting, and policy evaluation. Clearly this area is an extremely rich one in which much has been accomplished and much more will be accomplished in the future.

The great diversity of uses and results in the area of economic and econometric models can perhaps be underscored by mentioning some of the issues that have not been treated or treated only briefly, most of which are discussed elsewhere in this Handbook. These issues include, among others:

- Adaptive expectations
- Aggregation
- Asymptotic results
- Autoregressive moving average (ARMA) models
- Bayesian estimation
- Causality
- Certainty equivalence
- Computer simulation
- Disequilibrium models
- Distributed lags
- Dynamic multipliers
- Dynamic simulation
- Errors in variables models
- Exact finite sample results
- Expectations
- Functional forms for relationships
- Identification
- Lag structures
- Latent variables
- Limited dependent variables
- Matrix transition models
- Measurement errors
- Model simplicity/complexity
- Optimal control
- Partial adjustment models
- Path analysis
- Pooling cross-section and time-series data
- Qualitative economics
- Qualitative variables
- Random coefficients model
- Rational expectations
- Residual analysis
- Robust estimation
- Seasonality
- Seemingly unrelated equations
- Sequential hypothesis testing
- Specification error
- Spectral analysis
- Stochastic equilibrium
- Structural change
- Testing
- Time-varying parameters
- Unobserved variables

References

Adelman, I and F. L. Adelman (1959) "The Dynamic Properties of the Klein–Goldberger Model", *Econometrica*, 27, 596–625.

Agarwala, R. (1970) *An Econometric Model of India, 1948–61*. London: Frank Cass & Co.

Aitchison, J. and I. R. Dunsmore (1975) *Statistical Prediction Analysis*. Cambridge: Cambridge University Press.

Ando, A. and F. Modigliani (1969) "Econometric Analyses of Stabilization Policies", *American Economic Review*, 59, 296–314.

Ando, A., F. Modigliani and R. Rasche (1972) "Equations and Definitions of Variables for the FRB–MIT–Penn Econometric Model, November, 1969", in: B. G. Hickman (ed.), *Economic Models of Cyclical Behavior*. National Bureau of Economic Research. New York: Columbia University Press.

Ando, A. and C. Palash (1976) "Some Stabilization Problems of 1971–1975, with an Application of Optimal Control Algorithms", *American Economic Review*, 66, 346–348.

Apostol, T. (1974) *Mathematical Analysis* (2nd edn.). Reading Mass.: Addison-Wesley Publishing Co.

Arnold, S. F. (1981) *The Theory of Linear Models and Multivariable Analysis*. New York: John Wiley & Sons, Inc.

Arrow, K. J., H. B. Chenery, B. S. Minhas and R. M. Solow (1961) "Capital–Labor Substitution and Economic Efficiency", *Review of Economics and Statistics*, 43, 225–235.

Ball, R. J. (1968) "Econometric Model Building," in: *Mathematical Model Building in Economics and Industry*. London: Charles Griffen & Co., Ltd.

Ball, R. J. (ed.) (1973) *The International Linkage of Econometric Models*. Amsterdam: North-Holland Publishing Company.

Ball, R. J. and T. Burns (1968) "An Econometric Approach to Short-Run Analysis of the United Kingdom Economy, 1955–1966", *Operational Research Quarterly*, 19, 225–256.

Barten, A. P. (1977) "The Systems of Consumer Demand Functions Approach: A Review", in: M. D. Intriligator (ed.), *Frontiers of Quantitative Economics*, vol. III. Amsterdam: North-Holland Publishing Co.

Beach, E. F. (1957) *Economic Models: An Exposition*. New York: John Wiley & Sons, Inc.

Belsley, D. A. (1979) "On the Computational Competitiveness of FIML and 3SLS in the Estimation of Nonlinear Simultaneous-Equation Models", *Journal of Econometrics*, 9, 315–342.

Belsley, D. A. (1980) "On the Efficient Computation of the Nonlinear FIML Estimator", *Journal of Econometrics*, 14, 203–225.

Bergstrom, A. R. (1967) *Selected Economic Models and Their Analysis*. New York: American Elsevier Publishing Co., Inc.

Bowden, R. (1973) "The Theory of Parametric Identification", *Econometrica*, 41, 1069–1074.

Box, G. E. P. and D. R. Cox (1964) "An Analysis of Transformations", *Journal of the Royal Statistical Society*, B, 26, 211–243.

Box, G. E. P. and G. M. Jenkins (1970) *Time Series Analysis: Forecasting and Control*. San Francisco: Holden-Day.

Bridge, J. L. (1971) *Applied Econometrics*. Amsterdam: North-Holland Publishing Co.

Brown, J. A. C. and A. S. Deaton (1972) "Surveys in Applied Economics: Models of Consumer Behavior", *Economic Journal*, 82, 1143–1236.

Brown, M. (ed.) (1967) *The Theory and Empirical Analysis of Production*. National Bureau of Economic Research. New York: Columbia University Press.

Brown, M. and J. S. de Cani (1963) "Technological Change and the Distribution of Income", *International Economic Review*, 4, 289–309.

Brown, T. M. (1960) *Specification and Uses of Econometric Models*. London: Macmillan & Co., Ltd.

Chow, G. (1973) "On the Computation of Full Information Maximum Likelihood Estimates for Nonlinear Equation Systems", *Review of Economics and Statistics*, 55, 104–109.

Chow, G. (1975) *Analysis and Control of Dynamic Economic Systems*. New York: John Wiley & Sons, Inc.

Chow, G. (1981) *Econometric Analysis by Control Methods*. New York: John Wiley & Sons, Inc.

Christ, C. (1966) *Econometric Models and Methods*. New York: John Wiley & Sons, Inc.

Christ, C. (1975) "Judging the Performance of Econometric Models of the U.S. Economy", *International Economic Review*, 16, 54–74.
Christensen, L. R., D. W. Jorgenson and L. J. Lau (1973) "Transcendental Logarithmic Production Frontiers", *Review of Economics and Statistics*, 55, 28–45.
Cooper, R. L. (1972) "The Predictive Performance of Quarterly Econometric Models of the United States", in: B. G. Hickman (ed.), *Econometric Models of Cyclical Behavior*. New York: Columbia University Press.
Cramer, J. S. (1969) *Empirical Econometrics*. Amsterdam: North-Holland Publishing Co.
Data Resources, Inc. (1976) *The Data Resources National Economic Information System*. Amsterdam: North-Holland Publishing Co.
Deaton, A. S. (1975) *Models and Projections of Demand in Post-War Britain*. London: Chapman and Hall; New York: Halsted Press.
de Leeuw, F. and E. M. Gramlich (1968) "The Federal Reserve–MIT Econometric Model", *Federal Reserve Bulletin*, 54, 11–40.
de Leeuw, F. and E. M. Gramlich (1969) "The Channels of Monetary Policy: A Further Report on the Federal Reserve–MIT Model", *Journal of Finance*, 24, 265–290.
Douglas, P. H. (1948) "Are There Laws of Production?", *American Economic Review*, 38, 1–49.
Dhrymes, P. J. (1981) *Distributed Lags* (revised edn.). Amsterdam: North-Holland Publishing Company.
Duesenberry, J. S., G. Fromm, L. R. Klein and E. Kuh (eds.) (1965) *The Brookings Quarterly Econometric Model of the United States*. Chicago: Rand-McNally and Company.
Duesenberry, J. S., G. Fromm, L. R. Klein and E. Kuh (eds.) (1969) *The Brookings Model: Some Further Results*. Chicago: Rand-McNally & Company.
Duggal, V. G., L. R. Klein and M. D. McCarthy (1974) "The Wharton Model Mark III: A Modern IS–LM Construct", *International Economic Review*, 15, 572–594.
Eckstein, O. (ed.) (1976) *Parameters and Policies in the U.S. Economy*. Amsterdam: North-Holland Publishing Co.
Eckstein, O., E. W. Green and A. Sinai (1974) "The Data Resources Model: Uses, Structure, and Analysis of the U.S. Economy", *International Economic Review*, 15, 595–615.
Enke, S. (1951) "Equilibrium among Spatially Separated Markets: Solution by Electric Analogue", *Econometrica*, 19, 40–47.
Evans, M. K. (1969a) *Macroeconomic Activity: Theory, Forecasting and Control: An Econometric Approach*. New York: Harper & Row.
Evans, M. K. (1969b) *An Econometric Model of the French Economy: A Short-Term Forecasting Model*. Paris: Organization for Economic Cooperation and Development.
Evans, M. K. (1970) "An Econometric Model of the Israeli Economy 1952–1953", *Econometrica*, 38, 624–660.
Evans, M. K. (1974) "Econometric Models", in: W. F. Butler, R. A. Kavesh and R. B. Platt (eds.), *Methods and Techniques of Business Forecasting*. Englewood Cliffs, N.J.: Prentice-Hall, Inc.
Evans, M. K. and L. R. Klein (1967) *The Wharton Econometric Forecasting Model*. Philadelphia: Economics Research Unit, Wharton School, University of Pennsylvania.
Evans, M. K. and L. R. Klein (1968) *The Wharton Econometric Forecasting Model* (2nd enlarged edn.). Philadelphia: Economics Research Unit, Wharton School, University of Pennsylvania.
Evans, M. K., L. R. Klein and M. Saito (1972) "Short Run Prediction and Long Run Simulation of the Wharton Model", in: B. G. Hickman (ed.), *Econometric Models of Cyclical Behavior*. National Bureau of Economic Research. New York: Columbia University Press.
Fair, R. C. (1971) *A Short-Run Forecasting Model of the United States Economy*. Lexington: Heath Lexington Books.
Fair, R. C. (1974) "An Evaluation of a Short-Run Forecasting Model", *International Economic Review*, 15, 285–303.
Fair, R. C. (1976) *A Model of Macroeconomic Activity, Volume II: The Empirical Model*. Cambridge: Ballinger Publishing Company.
Fair, R. C. (1978) "The Use of Optimal Control Techniques to Measure Economic Performance", *International Economic Review*, 19, 289–309.
Fair, R. C. (1979) "An Analysis of the Accuracy of Four Macroeconometric Models", *Journal of Political Economy*, 87, 701–718.

Fair, R. C. and W. R. Parke (1980) "Full-Information Estimates of a Nonlinear Macroeconometric Model", *Journal of Econometrics*, 13, 269–291.
Ferguson, C. E. (1969) *The Neoclassical Theory of Production and Distribution*. New York: Cambridge University Press.
Fisher, F. M. (1966) *The Identification Problem in Econometrics*. New York: McGraw-Hill Book Company.
Fox, K. A., J. K. Sengupta and E. Thorbecke (1966) *The Theory of Quantitative Economic Policy, with Applications to Economic Growth and Stabilization*. Amsterdam: North-Holland Publishing Co.
Frisch, R. (1965) *Theory of Production*. Dordrecht: Reidel; Chicago: Rand-McNally.
Fromm, G. (ed.) (1971) *Tax Incentives and Capital Spending*. Amsterdam: North-Holland Publishing Co.
Fromm, G. and L. R. Klein (1973) "A Comparison of Eleven Econometric Models of the United States", *American Economic Review*, 63, 385–393.
Fromm, G. and L. R. Klein (eds.) (1975) *The Brookings Model: Perspective and Recent Developments*. Amsterdam: North-Holland Publishing Co.
Fromm, G. and P. Taubman (1968) *Policy Simulations with an Econometric Model*. Washington, D.C.: Brookings Institution.
Gallant, A. R. and D. W. Jorgenson (1979) "Statistical Inference for a System of Simultaneous, Nonlinear, Implicit Equations in the Context of Instrumental Variable Estimation", *Journal of Econometrics*, 11, 275–302.
Gallant, A. R. and A. Holly (1980) "Statistical Inference in an Implicit, Nonlinear, Simultaneous Equation Model in the Context of Maximum Likelihood Estimation", *Econometrica*, 48, 901–929.
Goldberger, A. S. (1959) *Impact Multipliers and Dynamic Properties of the Klein–Goldberger Model*. Amsterdam: North-Holland Publishing Co.
Goldberger, A. S. and O. D. Duncan (eds.) (1973) *Structural Equation Models in the Social Sciences*. New York: Seminar Press.
Goldfeld, S. M. and R. E. Quandt (1968) "Nonlinear Simultaneous Equations: Estimation and Prediction", *International Economic Review*, 9, 113–146.
Goldfeld, S. M. and R. E. Quandt (1972) *Nonlinear Methods in Econometrics*. Amsterdam: North-Holland Publishing Co.
Goldfeld, S. M. and R. E. Quandt (eds.) (1976) *Studies in Nonlinear Estimation*. Cambridge: Ballinger Publishing Co.
Granger, C. W. J. and P. Newbold (1977) *Forecasting Economic Time Series*. New York: Academic Press.
Griliches, Z. (1967) "Distributed Lags: A Survey", *Econometrica*, 35, 16–49.
Haitovsky, Y., G. Treyz and V. Su (1974) *Forecasts with Quarterly Macroeconometric Models*. National Bureau of Economic Research. New York: Columbia University Press.
Helliwell, J. F., L. H. Officer, H. T. Shapiro and J. A. Stewart (1969) *The Structure of RDX1*. Ottawa: Bank of Canada.
Helliwell, J. F., H. T. Shapiro, G. R. Sparks, I. A. Stewart, F. W. Gerbet and D. R. Stevenson (1971) *The Structure of RDX2*. Ottawa: The Bank of Canada.
Hendry, D. F. (1974) "Stochastic Specification in an Aggregate Demand Model of the United Kingdom", *Econometrica*, 42, 559–578.
Hickman, B. G. (ed.) (1965) *Quantitative Planning of Economic Policy*. Washington, D.C.: The Brookings Institution.
Hickman, B. G. (ed.) (1972) *Econometric Models of Cyclical Behavior*. National Bureau of Economic Research. New York: Columbia University Press.
Hilton, K. and D. F. Heathfeld (eds.) (1970) *The Econometric Study of the United Kingdom; Proceedings of the 1969 Southampton Conference on Short-Run Econometric Models of the U.K. Economy*. London: Macmillan.
Hoffman, K. (1975) *Analysis in Euclidean Space*. Englewood Cliffs, N.J.: Prentice-Hall, Inc.
Houthakker, H. S. (1957) "An International Comparison of Household Expenditure Patterns Commemorating the Centenary of Engel's Laws", *Econometrica*, 25, 532–551.
Houthakker, H. S. (1965) "New Evidence on Demand Elasticities", *Econometrica*, 33, 277–288.
Howrey, E. P. (1972) "Dynamic Properties of a Condensed Version of the Wharton Model", in: B. G. Hickman (ed.), *Econometric Models of Cyclical Behavior*. New York: Columbia University Press.

Ichimura, S., L. R. Klein, S. Koizumi, K. Sato and Y. Shinkai (1964) "A Quarterly Econometric Model of Japan, 1952–59", *Osaka Economic Papers*, 12, 19–44.
Intriligator, M. D. (1971) *Mathematical Optimization and Economic Theory*. Englewood Cliffs, N.J.: Prentice-Hall, Inc.
Intriligator, M. D. (1978) *Econometric Models, Techniques, and Applications*. Englewood Cliffs, N.J.: Prentice-Hall, Inc.; Amsterdam: North-Holland Publishing Co.
Intriligator, M. D. (1981) "Mathematical Programming, with Applications to Economics", in: K. J. Arrow and M. D. Intriligator (eds.), *Handbook of Mathematical Economics*, Vol. I. Amsterdam: North-Holland Publishing Co.
Jorgenson, D. W. and J. J. Laffont (1974) "Efficient Estimation of Nonlinear Simultaneous Equations with Additive Disturbances", *Annals of Economic and Social Measurement*, 3, 615–640.
Kalman, P. J. and M. D. Intriligator (1973) "Generalized Comparative Statics, with Applications to Consumer and Producer Theory", *International Economic Review*, 14, 473–486.
Kendall, M. G. (1968) "Introduction to Model Building and its Problems", in: *Mathematical Model Building in Economics and Industry*. London: Charles Griffen & Co., Ltd.
Klein, L. R. (1950) *Economic Fluctuations in the United States, 1921–1941*. Cowles Commission Monograph No. 11. New York: John Wiley & Sons, Inc.
Klein, L. R. (1971) "Forecasting and Policy Evaluation using Large-Scale Econometric Models: The State of the Art", in: M. D. Intriligator (ed.), *Frontiers of Quantitative Economics*. Amsterdam: North-Holland Publishing Co.
Klein, L. R. (1977) "Economic Policy Formation through the Medium of Econometric Models", in: M. D. Intriligator (ed.), *Frontiers of Quantitative Economics*, Vol. III. Amsterdam: North-Holland Publishing Co.
Klein, L. R., R. J. Ball, A. Hazlewood and P. Vandome (1961) *An Econometric Model of the United Kingdom*. Oxford: Basil Blackwell.
Klein, L. R. and E. Burmeister (eds.) (1976) *Econometric Model Performance: Comparative Simulation Studies of the U.S. Economy*. Philadelphia: University of Pennsylvania Press.
Klein, L. R. and A. S. Goldberger (1955) *An Econometric Model of the United States, 1929–1952*. Amsterdam: North-Holland Publishing Co.
Klein, L. R. and Y. Shinkai (1963) "An Econometric Model of Japan, 1930–59", *International Economic Review*, 4, 1–28.
Klein, L. R. and R. M. Young (1980) *An Introduction to Econometric Forecasting Models*. Lexington: Lexington Books.
Kmenta, J. (1967) "On the Estimation of the CES Production Function", *International Economic Review*, 8, 180–189.
Kosobud, R. and R. Minami (eds.) (1977) *Econometric Studies of Japan*. Urbana: University of Illinois Press.
Leamer, E. E. (1978) *Specification Searches*. New York: John Wiley & Sons, Inc.
Lucas, R. E. and T. J. Sargent (eds.) (1980) *Rational Expectations and Econometric Practice*. Minneapolis: University of Minnesota Press.
Maddala, G. S. (1977) *Econometrics*. New York: McGraw-Hill Book Co.
Malinvaud, E. (1970) *Statistical Methods of Econometrics* (2nd rev. edn.). Amsterdam: North-Holland Publishing Co.
Marschak, J. and W. H. Andrews (1944) "Random Simultaneous Equations and the Theory of Production", *Econometrica*, 12, 143–205.
McCarthy, M. D. (1972) *The Wharton Quarterly Econometric Forecasting Model, Mark III*. Philadelphia: Economics Research Unit, University of Pennsylvania.
Minhas, B. S. (1963) *An International Comparison of Factor Costs and Factor Use*. Amsterdam: North-Holland Publishing Co.
Morehouse, N. F., R. H. Strotz and S. J. Horwitz (1950) "An Electro-Analog Method for Investigating Problems in Economic Dynamics: Inventory Oscillations", *Econometrica*, 18, 313–328.
Muench, T., A. Rolneck, N. Wallace and W. Weiler (1974) "Tests for Structural Change and Prediction Intervals for the Reduced Form of Two Structural Models of the U.S.: The FRB–MIT and Michigan Quarterly Models", *Annals of Economic and Social Measurement*, 3, 491–520.
Naylor, T. H. (1971) "Policy Simulation Experiments With Macroeconometric Models: The State of the Art", in: M. D. Intriligator (ed.), *Frontiers of Quantitative Economics*. Amsterdam: North-Holland Publishing Co.

Naylor, T. H., K. Wertz and T. Wonnacott (1968) "Some Methods For Evaluating the Effects of Economic Policies using Simulation Experiments", *Review of the International Statistical Institute*, 36, 184–200.
Nerlove, M. (1965) *Estimation and Identification of Cobb–Douglas Production Functions*. Amsterdam: North-Holland Publishing Co.
Nerlove, M. (1966) "A Tabular Survey of Macroeconometric Models", *International Economic Review*, 7, 127–175.
Nerlove, M., D. M. Grether and J. L. Carvalho (1979) *Analysis of Economic Time Series*. New York: Academic Press.
Nicholls, D. F., A. R. Pagan and R. D. Terrell (1975) "The Estimation and Use of Models with Moving Average Disturbance Terms: A Survey", *International Economic Review*, 16, 113–134.
Palm, F. (1977) "On Univariate Time Series Methods and Simultaneous Equation Econometric Models", *Journal of Econometrics*, 5, 379–388.
Phlips, L. (1974) *Applied Consumption Analysis*. Amsterdam: North-Holland Publishing Co.
Pindyck, R. S. (1973) *Optimal Planning for Economic Stabilization*. Amsterdam: North-Holland Publishing Co.
Pindyck, R. S. and D. L. Rubinfeld (1976) *Econometric Models and Economic Forecasts*. New York: McGraw-Hill Book Co.
Pollak, R. A. and T. J. Wales (1969) "Estimation of the Linear Expenditure System", *Econometrica*, 37, 611–628.
Powell, A. A. (1974) *Empirical Analytics of Demand Systems*. Lexington: Lexington Books.
Prais, S. J. and H. S. Houthakker (1955) *The Analysis of Family Budgets*. New York: Cambridge University Press.
Preston, R. S. (1972) *The Wharton Annual and Industry Forecasting Model*. Philadelphia: Economics Research Unit, Wharton School, University of Pennsylvania.
Preston, R. S. (1975) "The Wharton Long-Term Model: Input–Output Within the Context of a Macro Forecasting Model", *International Economic Review*, 16, 3–19.
Quirk, J. and R. Saposnik (1968) *Introduction to General Equilibrium Theory and Welfare Economics*. New York: McGraw-Hill Book Company.
Rasche, R. H. and H. T. Shapiro (1968) "The FRB–MIT Econometric Model: Its Special Features and Implications for Stabilization Policies", *American Economic Review*, 58, 123–149.
Renton, G. A., Ed. (1975) *Modelling the Economy*. London: Heinemann.
Rothenberg, T. J. (1971) "Identification in Parametric Models", *Econometrica*, 38, 577–591.
Rothenberg, T. J. (1973) *Efficient Estimation with A Priori Information*. Cowles Foundation Monograph 23, New Haven: Yale University Press.
Rudin, W. (1964) *Principles of Mathematical Analysis*. New York: McGraw-Hill Book Company.
Samuelson, P. A. (1947) *Foundations of Economic Analysis*. Cambridge: Harvard University Press.
Samuelson, P. A. (1975) "The Art and Science of Macro Models", in: G. Fromm and L. R. Klein (eds.), *The Brookings Model: Perspective and Recent Developments*. Amsterdam: North-Holland Publishing Co.
Sargent, T. J. (1981) "Interpreting Economic Time Series", *Journal of Political Economy*, 89, 213–248.
Shapiro, H. T. and L. Halabuk (1976) "Macro-Econometric Model Building in Socialist and Non-Socialist Countries: A Comparative Study", *International Economic Review*, 89, 213–248.
Silberberg, E. (1978) *The Structure of Economics: A Mathematical Analysis*. New York: McGraw-Hill Book Company.
Sims, C. A. (1974) "Distributed Lags", in: M. D. Intriligator and D. A. Kendrick (eds.), *Frontiers of Quantitative Economics*, vol. II. Amsterdam: North-Holland Publishing Co.
Smith, A. (1776) *The Wealth of Nations*, Edited by Edwin Cannan (1937). New York: The Modern Library.
Stone, R. (1954) *The Measurement of Consumers' Expenditure and Behavior in the United Kingdom, 1920–1938*. New York: Cambridge University Press.
Stone, R. (1972) *A Computable Model of Economic Growth: A Programme for Growth*, Vol. 1. Cambridge: Chapman and Hall.
Stone, R., A. Brown and D. A. Rowe (1965) "Demand Analysis and Projections in Britain: 1900–1970", in: J. Sandee (ed.), *Europe's Future Consumption*. Amsterdam: North-Holland Publishing Co.

Strotz, R. H., J. C. McAnulty and J. B. Naines, Jr. (1953) "Goodwin's Nonlinear Theory of the Business Cycle: An Electro-Analog Solution", *Econometrica*, 21, 390–411.

Suits, D. B. (1962) "Forecasting and Analysis with an Econometric Model", *American Economic Review*, 52, 104–132.

Suits, D. B. (1963) *The Theory and Application of Econometric Models*. Athens: Center of Economic Research.

Suits, D. B. (1964) *An Econometric Model of the Greek Economy*. Athens: Center of Economic Research.

Sylos-Labini, P. (1974) *Trade Unions, Inflation and Productivity*. Lexington: Lexington Books.

Theil, H. (1961) *Economic Forecasts and Policy* (2nd edn.). Amsterdam: North-Holland Publishing Co.

Theil, H. (1964) *Optimal Decision Rules for Government and Industry*. Chicago: Rand-McNally & Co.; Amsterdam: North-Holland Publishing Co.

Theil, H. (1966) *Applied Economic Forecasting*. Amsterdam: North-Holland Publishing Co.

Theil, H. (1971) *Principles of Econometrics*. New York: John Wiley & Sons, Inc.

Theil, H. (1975/1976) *The Theory and Measurement of Consumer Demand*, Vols. I and II. Amsterdam: North-Holland Publishing Co.

Theil, H. and J. C. G. Boot (1962) "The Final Form of Econometric Equation Systems", *Review of the International Statistical Institute*, 30, 136–152.

Tinbergen, J. (1955) *On the Theory of Economic Policy* (2nd edn.). Amsterdam: North-Holland Publishing Co.

Tinbergen, J. (1956) *Economic Policy: Principles and Design*. Amsterdam: North-Holland Publishing Co.

Tustin, A. (1953) *The Mechanism of Economic Systems*. Cambridge: Harvard University Press.

Ueno, H. (1963) "A Long-Term Model of the Japanese Economy, 1920–1958", *International Economic Review*, 4, 171–193.

Walters, A. A. (1963) "Production and Cost Functions: An Econometric Survey", *Econometrica*, 31, 1–66.

Walters, A. A. (1968) "Econometric Studies of Production and Cost Functions", *Encyclopedia of the Social Sciences*.

Wold, H. (1954) "Causality and Econometrics", *Econometrica*, 22, 162–177.

Wold, H. (1960) "A Generalization of Causal Chain Models", *Econometrica*, 28, 443–463.

Wold, H. (1968) *Econometric Model Building: Essays on the Causal Chain Approach*. Amsterdam: North-Holland Publishing Co.

Wold, H. and L. Jureen (1953) *Demand Analysis*. New York: John Wiley & Sons, Inc.

Zarnowitz, V. (1967) *An Appraisal of Short-Term Economic Forecasts*. New York: National Bureau of Economic Research.

Zellner, A. (1979) "Statistical Analysis of Econometric Models", *Journal of the American Statistical Association*, 74, 628–643, with Comments by D. A. Belsley and E. Kuh (643–645), C. F. Christ (645–646), P. M. Robinson (646–648), T. J. Rothenberg (648–650), and Rejoinder of A. Zellner (650–651).

Zellner, A., J. Kmenta and J. Drèze (1966) "Specification and Estimation of Cobb–Douglas Production Function Models", *Econometrica*, 34, 727–729.

Zellner, A. and F. Palm (1974) "Time Series Analysis and Simultaneous Equation Econometric Models", *Journal of Econometrics*, 2, 17–54.